The Effect of Profit Sharing on Auction Markets[*]

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Current version: November 15, 2004

Abstract. We consider an independent private value auction environment in which bidders own passive partial claims on rival bidder's auction profits. While cross ownership confers no ability to directly affect bidding behavior, claims on rival profits dampen competition. When bidders hold the same aggregate level of shares in rivals in a first-price auction, cross ownership has an effect similar to reducing the number of bidders. A similar decrease in competition occurs in English auctions. In contrast to the well-known revenue equivalence result, when bidders own the same shares in all rival bidders, the seller prefers first-price auctions to English auctions.

[*]A version of this research was presented at the 2004 Winter Meetings of the Econometric Society in San Diego. We thank Sudipto Dasgupta, David Ettinger, Sheldon Kimmel, Leslie Marx, Rob Porter, and Chuck Thomas for useful comments. The views expressed herein are solely those of the authors and do not represent the views of the United States Department of Justice.

1. Introduction

Firms often own partial equity interests in other firms that are horizontal competitors, vertically-related, or both. Examples include Northwest Airlines investing in Continental Airlines (air travel), Microsoft in Apple (computer operating systems), TCI in Time Warner (cable television programming and distribution), Gillette in Wilkinson Sword (wet-shaving razor blades), and Dairy Farmers of America in Southern Belle Dairy (school milk).[1] Typically these stakes amount to minority ownership positions that convey a share of firm profits but little or no firm control. Bresnahan and Salop (1986) label such passive stakes "silent financial interests." Competition authorities frequently review acquisitions of such interests.

While some work on partial ownership has addressed efficiency rationales for partial ownership, especially in vertical contexts,[2] much of the literature has focused on the competition-reducing effects of partial ownership among horizontal competitors. In contrast to a large literature examining cross ownership in Cournot settings,[3] as well as an emerging literature on merger effects in auction environments,[4] only three other papers address cross ownership among bidders: Dasgupta and Tsui (2004) and Ettinger (2002, 2003).[5] The present analysis contributes to this recent body of work by generalizing from the two bidder case to n bidders,[6] by providing intuition for how the level of cross ownership alters the number of effective bidders, and by offering a novel proof for ranking first-price and English auctions in terms of seller revenue. Our model also bears some similarity to the bidding with externalities settings examined by Jehiel *et al.* (1996) and Jehiel and Moldovanu (1996). In that literature, however, the size of the externality does not depend on the sale price, while in our setting higher prices diminish profit streams to the winning bidder and all of its shareholders.

Using the terminology of the Merger Guidelines issued by the U.S. Department of Justice and the U.S. Federal Trade Commission, we focus in the present analysis on unilateral effects rather than coordinated effects. That is, we do not investigate the extent to which partial

[1] See Gilo (2000) for a discussion of most of these examples and others, as well as an analysis of how the "solely for investment" exception in section 7 of the Clayton Act has been applied to passive investments in rival firms. In April 2003, the U.S. Department of Justice filed a civil lawsuit to compel Dairy Farmers of America Inc. to divest its ownership interest in Southern Belle.

[2] See, for example, Riordan (1991), Dasgupta and Tao (2000), and Greenlee and Raskovich (2004).

[3] This literature is extensive. See, for example, Bresnahan and Salop (1986), Farrell and Shapiro (1990), Flath (1991), Gilo (2000) and cites therein.

[4] See, for example, Brannman and Froeb (2000), Waehrer and Perry (2003), and cites therein.

[5] While focusing on the two-bidder case, Dasgupta and Tsui employ a value function more general than the independent private values setting examined here. In addition to horizontal cross ownership, Ettinger (2002) considers partial vertical ownership–namely, ownership of the seller by bidders. Ettinger (2003) focuses on efficiency considerations and demonstrates that in the presence of cross ownership, the second price auction and English are not equivalent.

[6] Ettinger (2003) considers n bidders, but allows only two bidders to have symmetric holdings in each other. In his framework, all other bidders own no stakes in rivals and remain privately held.

cross ownership may promote bidder collusion.[7] Our auction results generally coincide with the previous literature on horizontal cross ownership. Concern about cannibalizing profit streams flowing from rival bidders causes firms to bid less aggressively. In the limit as each bidder places the same importance on each of its rival's operational profits as on its own, the bidding equilibrium approaches the outcome one would observe following a merger of all bidders, or the formation of an industry-wide perfect cartel.

The collusion analogy goes a step further. We compare (expected) equilibrium prices for first-price and English auctions in independent private values environments, and find that given positive and symmetric levels of cross ownership, English auctions suffer larger anticompetitive price effects than first-price auctions. That is, faced with cross ownership among bidders, sellers strictly prefer first-price auctions to English auctions. In contrast to other auction ranking results, bidders in our setting earn the same informational rents in first-price and English auctions. A difference in the expected payoff to a bidder with the lowest possible value in the two auction forms drives our result. This revenue ranking coincides with work establishing that English and second-price auctions are more prone to collusion than first-price auctions.[8] Our result, however, arises purely from noncoordinated behavior.

The balance of this paper is organized in the following manner. Section 2 presents the basic bidding framework, and introduces the possibility for partial ownership of rival firms. Focusing on independent private values environments, Section 3 examines equilibrium bidding behavior in first-price auctions and English auctions and ranks the two auction formats in terms of expected seller revenue. Section 4 briefly concludes and an appendix contains proofs of propositions.

2. Model

Let $N = \{1, \ldots, n\}$, where $n > 1$, denote the set of risk-neutral firms competing to purchase an item.[9] Let $\mathbf{v} = (v_1, \ldots, v_n)$ and $\mathbf{b} = (b_1, \ldots, b_n)$ respectively denote the vector of firm values and bids. We study a symmetric, independent private values environment. Namely, each bidder knows its own value for the object but views each rival's value as an independently and identically distributed draw from the distribution function F with continuous density f over the support $[\underline{v}, \overline{v}]$.

[7] Cross ownership may facilitate collusion by providing a mechanism for sharing revenue, improving information transmission between bidders, or reducing the incentive for bidders to submit additional (competitive) bids outside a cartel agreement.

[8] For general discussions of collusion in auctions, consult Hendricks and Porter (1989) and Froeb and Shor (2000).

[9] While we present the model as bidders competing as buyers, the results also apply to the case where bidders compete to supply an item.

Let $\Pi_i(\mathbf{v}, \mathbf{b})$ denote firm i's expected profit when it bids b_i, rival bidders $j \in N\backslash\{i\}$ bid b_j, and there is no cross ownership. The expression for this expected profit depends on the type of auction conducted but it is essentially the bidder's value minus the expected price conditional on the bidder winning all multiplied by the probability of the bidder winning. We refer to this as firm i's operational profits. When there is cross ownership among firms, each manager adopts a bidding strategy to maximize the firm's total returns, taking into account any profit streams it receives as a partial owner of rival bidders. Letting α_{ij} denote the weight that firm i management places on the operational profits of firm j, firm i seeks to maximize

$$U_i(\mathbf{v}, \mathbf{b}) = \sum_{j \in N} \alpha_{ij} \Pi_j(\mathbf{v}, \mathbf{b}). \tag{1}$$

While we refer to α_{ij} as firm i's ownership share of firm j, there are alternative ways in which these weights can be generated from joint ownership. We discuss this issue in the Appendix.

Like much of the auction literature, we make certain symmetry assumptions without which analytic solutions are often impossible to derive. The symmetry assumption also serves to create an environment where we might expect revenue equivalence to hold.

Definition 1. *Let $\bar{\alpha}_i \equiv \frac{1}{n-1} \sum_{j \in N\backslash\{i\}} \alpha_{ij}$. (i) Ownership shares are* ***symmetric*** *if and only if for all $i, j \in N$, $\alpha_{ii} = \alpha_{jj}$, and for all $k \neq i$ and $h \neq j$, $\alpha_{ik} = \alpha_{jh}$. (ii) Ownership shares are* ***semi-symmetric*** *if and only if for all $i, j \in N$, $\alpha_{ii} = \alpha_{jj}$ and $\bar{\alpha}_i = \bar{\alpha}_j$.*

If for all i, the ith row of A is the vector of weights in firm i's objective function, then ownership shares are symmetric if and only if all diagonal elements of A are equal and all off-diagonal elements are equal. Semi-symmetry requires the diagonal elements of A to be equal, but otherwise only requires that each row sum to the same value.

We assume throughout that a firm's own operational profits play an important role in determining bid levels. Specifically, we require that $\alpha_{ii} > \bar{\alpha}_i \geqslant 0$ for all i, implying that a bidder places more weight on its own operational profits than it does (on average) on each rival's operational profits.[10]

3. First-price and English Auctions

In a first-price auction, bidders simultaneously submit bids, the highest bidder wins the object and pays the amount of its own bid if its bid exceeds the reserve price r. We assume

[10] Observe that each firm's maximization problem is invariant to a rescaling of the weights included in its objective function. As long as $\alpha_{ii} > 0$ for all i, one may assume without loss of generality that $\alpha_{ii} = \alpha_{jj}$ for all i, j, and thus the symmetry definitions can be viewed as restrictions just on the off-diagonal elements of A.

that $r \in [\underline{v}, \overline{v})$. When each firm's value is private information, a bidding strategy is a function that maps each possible value to a bid $b : [\underline{v}, \overline{v}] \to \mathbb{R}$. For tractability, we focus on semi-symmetric ownership profiles and determine the symmetric equilibrium bidding strategy. Let $b^F(\cdot)$ denote the bidding strategy followed by bidder i's rivals, and assume that it is an increasing function of the bidder's value. Define the function ϕ^F to be the inverse of b^F. Given v_i, firm i's management sets β to maximize

$$\begin{aligned} u_i^F(v_i, \beta) &= \alpha_{ii} \left[v_i - \beta \right] F(\phi^F(\beta))^{n-1} \cdot 1_{\{\beta \geqslant r\}} \\ &+ \bar{\alpha}_i \int_{\max\{\phi^F(\beta), r\}}^{\overline{v}} [\xi - b^F(\xi)](n-1) F(\xi)^{n-2} f(\xi) d\xi. \end{aligned}$$

The first term is α_{ii} times firm i's expected operational profits while the second term is firm i's weighted expected profit stream from other bidders. When $\bar{\alpha}_i = 0$, this objective function simplifies to the standard payoff function for a bidder in a first-price auction.

Proposition 1. *Suppose ownership shares are semi-symmetric and let* $\omega = (n-1)\left(1 - \frac{\bar{\alpha}_i}{\alpha_{ii}}\right)$. *For a first-price sealed bid auction, bidders with* $v_i < r$ *choose not to bid,* $b^F(r) = r$ *and for* $v_i > r$,

$$b^F(v_i) = v_i - \int_r^{v_i} \frac{F(\xi)^\omega}{F(v_i)^\omega} d\xi$$

is an equilibrium bidding strategy.

Proof in the Appendix.[11]

The effect of cross ownership depends on the ratio of the average ownership share in rival bidders $\bar{\alpha}_i$ to a bidder's share in its own operation α_{ii}. The next results follow in a straightforward manner from the equilibrium bidding strategy described in Proposition 1.

Corollary 1. *For the first-price sealed bid auction with semi-symmetric ownership shares, (i) the equilibrium bidding strategy* b^F *is decreasing in* $\bar{\alpha}_i/\alpha_{ii}$, *(ii) if* $\bar{\alpha}_i = 0$, *then the equilibrium bidding strategy* b^F *is the equilibrium bidding strategy with no cross ownership, (iii) in the limit as* $\bar{\alpha}_i/\alpha_{ii}$ *approaches* 1 *for all* $i \in N$, *the equilibrium bidding strategy* $b^F(v)$ *approaches* r *for all* $v \in [r, \overline{v}]$.

Like partial ownership in Cournot environments, part (i) indicates that increasing the importance of rival firm operational profits in each firm's objective function dampens competition. Observe that the number of bidders n and cross ownership (α_{ii} and $\bar{\alpha}_i$) enter the equilibrium bidding strategy only via the exponent ω. This exponent can be thought

[11] Proofs of this result for the two bidder case can be found in Dasgupta and Tsui (2004) and Ettinger (2002).

of as measuring the number of effective rivals that each firm faces, and $\omega + 1$ as the total number of effective competitors. When there is no cross ownership, $\omega = n - 1$, and part (ii) indicates that the equilibrium bidding strategy matches the standard case. As ownership in rival bidders increases in importance ($\bar{\alpha}_i/\alpha_{ii}$ increases), ω and the equilibrium bidding strategy b^F decline. In the limit as $\bar{\alpha}_i$ approaches α_{ii}, the number of effective rivals to each firm approaches zero and part (iii) states that the corresponding bidding outcome matches the single bidder case. That is, in the limit all firms with values exceeding r submit the minimum acceptable bid.

We next turn to English auctions. In this bidding environment, starting at r, the price steadily rises and each bidder remains in the auction only until the price reaches a level at which that bidder chooses to drop out. The auction stops when only one bidder remains, the last bidder wins the object, and pays the price at which the penultimate bidder exited. If only one bidder is active at the reserve price, then the price is set equal to r. In a two bidder English auction, a bidding strategy is simply a price at which to exit the auction. The following result coincides with results in Dasgupta and Tsui (2004) and Ettinger (2002, 2003).[12]

Proposition 2. *Assume $N = \{1, 2\}$, $\alpha_{11} = \alpha_{22} > \alpha_{21} = \alpha_{12} > 0$, and let $\eta = \frac{\alpha_{11} - \alpha_{12}}{\alpha_{12}}$. Then bidders with $v_i \leqslant r$ choose not to bid and for $v_i > r$*

$$b^E(v_i) = v_i - \int_r^{v_i} \frac{[1 - F(v_i)]^\eta}{[1 - F(\xi)]^\eta} d\xi,$$

is an equilibrium bidding strategy in a two bidder English auction.

Proof in the Appendix.

English auctions with no cross ownership have a dominant strategy equilibrium. Since the winning bidder does not determine the final price, and losing bidders are indifferent to the level of the final price (provided there is no cross ownership), it is optimal for bidder i to remain in the auction until the price reaches v_i and then exit. That attractive feature of English auctions disappears when there is cross ownership among bidders. Owning stakes in rival bidders causes bidder i to care about the final price, even if it were to learn that it will not win the auction. For any $v_i < \overline{v}$, bidder i believes that it will ultimately lose the auction with positive probability. In such (losing) circumstances, it is costly for bidder i to remain active because doing so simply raises the expected auction price and reduces the profit stream bidder i receives from the winning bidder. When $v_i = \overline{v}$, fears that bidder i may only be raising the price paid by another bidder are not present, and the equilibrium bidding

[12] The results in Dasgupta and Tsui (2004) and Ettinger (2002) are stated as applying to second-price auctions, but in the two bidder case second-price and English auctions are strategically equivalent.

strategy coincides with the equilibrium bidding strategy when there is no cross ownership: $b^E(\overline{v}) = \overline{v}$. When there is cross ownership, bidders whose value is strictly smaller than $\overline{v}$ recognize this additional cost to remaining active, and choose to exit sooner. Given these considerations and the sequential nature of English auctions, calculating equilibria when there are more than two bidders is difficult. However, as we show below, it is still possible to compare the expected revenue generated by English and first-price auctions.

The equilibrium bidding function of Proposition 2 exhibits similar comparative statics as the first-price auction. Namely, prices decline as rival firm profits take on greater importance to each bidder, resulting in a complete absence of competition as α_{12} approaches α_{11} and α_{21} approaches α_{22}. In a fashion similar to the first-price auction case, each of the statements in the next corollary follows straightforwardly from the equilibrium bidding strategy in Proposition 2.

Corollary 2. *For an English auction with $N = \{1, 2\}$ and $\alpha_{11} = \alpha_{22} > \alpha_{21} = \alpha_{12} > 0$, (i) The equilibrium bidding strategy b^E is decreasing in α_{ij}/α_{ii}. (ii) In the limit as α_{ij} approaches zero, b^E matches the equilibrium bidding strategy with no cross ownership. Namely $b^E(v_i) = v_i$ for all i. (iii) In the limit as $\bar{\alpha}_i/\alpha_{ii}$ approaches 1 for all $i \in N$, the equilibrium bidding strategy $b^E(v)$ approaches r for all $v \in [r, \overline{v}]$.*

Since a firm's optimal bid in an English auction is independent of the number of bidders when there is no cross ownership, it is not possible to calculate a number of effective competitors as we did for the first-price auction. We can, however, compare the seller's expected revenue from using first-price and English auctions. For the two-bidder case, Corollaries 1 and 2 together imply that English auctions and first-price auctions generate the same expected seller revenue as α_{ij} approaches α_{ii} for all $i, j \in \{1, 2\}$. Namely, all competition is eliminated and the expected price, conditional on making a sale, approaches r. At the other extreme, standard revenue equivalence results imply a similar comparison when there is no cross ownership ($\alpha_{21} = \alpha_{12} = 0$).[13] The follow lemma and proposition establish an expected revenue ordering of the two auction mechanisms for general n and any symmetric ownership shares.[14]

Lemma 1. *Suppose auction forms X and Y each satisfy the following properties: (i) they employ the same reserve price, (ii) only the winning bidder makes a payment, and (iii) in equilibrium, the bidder with the highest value always wins the object, provided that*

[13] See, for example, Riley and Samuelson (1981).

[14] Marshall and Schulenberg (2003) find that revenue rankings that hold without any reserve price do not necessarily hold when the seller can set a reserve price optimally. In contrast, we find a definitive ranking of first-price and English auctions, even with optimal reserve prices.

its value exceeds the common reserve price. If X and Y satisfy these properties, then X generates higher seller revenue than Y if and only if the expected payoff to a bidder with value $\underline{v}$ is higher for Y than for X.

Proof in the Appendix.

Lemma 1 resembles part of a revenue equivalence result. The only remaining step needed to establish revenue equivalence of X and Y would be to show that the expected payoff to bidders with value $\underline{v}$ are equal (typically to zero) for X and for Y. In the presence of cross ownership, however, the expected payoff to bidders with value $\underline{v}$ is positive since they receive some share of the winner's expected revenue. The following proposition ranks first-price and English auctions by comparing the expected payoffs to bidders with value $\underline{v}$.

Proposition 3. *Suppose ownership shares are symmetric, $\alpha_{ii} > \bar{\alpha}_i > 0$, and consider equilibria where the bidder with the highest value always wins the auction, provided that its value exceeds the reserve price. Sellers prefer first-price auctions to English auctions when the same reserve price is used in the two auctions, or when the reserve price is set optimally in each auction.*

Proof in the Appendix.[15]

Letting $\widetilde{u}_i^{\mathrm{X}}(v_i, v_i')$ denote bidder i's expected payoff in auction X when bidder i has value v_i but bids as if it had value v_i', the proof of Lemma 1 establishes that

$$d\widetilde{u}_i^{\mathrm{X}}(v_i, v_i)/dv_i = \alpha_{ii} F(v_i)^{n-1}$$

for all auction forms X that satisfy the conditions of Lemma 1. If $\widetilde{u}_i^{\mathrm{X}}(v_i, v_i) - \widetilde{u}_i^{\mathrm{X}}(\underline{v}, \underline{v})$ are the informational rents that accrue to a bidder of type v_i, then this equation implies that bidders earn the same informational rents for all such auction forms X. Thus, differences in the expected payoff to a bidder with value $\underline{v}$ determine the auction forms that generate larger expected seller revenues. In a first-price auction, the price depends only on the highest value, and the competition that the winning bidder anticipates when formulating its bid. In contrast, the price in an English auction depends completely on the actual competitiveness of losing bidders. Thus, unlike in first-price auctions, the winning bidder strictly benefits from facing rivals with low values in English auctions. Given its partial ownership of the auction winner, a bidder with the lowest possible value benefits in a similar fashion.[16,17]

[15] Proofs of this result for the two bidder case can be found in Dasgupta and Tsui (2004) and Ettinger (2002).

[16] Observe that when there is no cross ownership, a bidder with value $\underline{v}$ earns the same expected profit (zero) in English and first-price auctions, and we obtain the standard revenue equivalence result.

[17] Dasgupta and Tsui (2004) make the same argument for the two bidder case. A similar effect does not occur in Jehiel *et al.* (1996) and Jehiel and Moldovanu (1996) because the externalities they consider do not depend on the profit level of the winning bidder.

Therefore, compared to English auctions, first-price auctions extract greater surplus from the bidder with the lowest type, and hence generate greater expected revenue for the seller.

Observe that Lemma 1 and Proposition 3 establish an expected revenue ranking by showing that first-price and English auctions result in the same informational rents to bidders but differ in the rents accruing to the bidder with the lowest possible value. In contrast most revenue rankings in symmetric settings follow from the Linkage Principle where the two auction forms differ in the size of information rents captured by bidders and not in the rents accruing to the bidder with value $\underline{v}$.[18]

4. Conclusion

We present a model of cross ownership, consider the resulting unilateral competitive effects, and find that cross ownership weakens competition between bidders. Bidders recognize that submitting high bids may cannibalize profit streams flowing from ownership stakes in rival bidders. In the limit as all bidders place the same importance on each firm's operational profits, all competition is eliminated and the bidding equilibrium resembles an industry-wide merger or perfect cartel. We also establish that English auctions suffer larger anticompetitive effects than first-price auctions.

For various competitive and informational environments, a large strand of the auction literature ranks different auction formats according to their expected seller revenue. Milgrom and Weber (1982), for example, establish that sellers prefer English auctions to first-price auctions in affiliated values settings. English auctions, however, are more susceptible to collusive behavior than first-price auctions, and as we establish here, suffer from greater anticompetitive effects in the presence of cross ownership among bidders. Therefore, in an affiliated values setting with cross ownership, the relative effectiveness of an English auction compared to a first-price auction will depend on whether the learning effects (inferring the signals of exiting bidders) exceed the anticompetitive effects (dampened incentive to bid aggressively). Taking these, along with similar rankings for other environments, the natural conclusion is that the optimal auction mechanism for a seller will vary on a case by case basis. Our results contribute to the factors a seller should consider in selecting an auction format.

[18] See Milgrom (1987) for a discussion of the Linkage Principle.

Appendix

Interpretation of weights: The weights included in Equation (1) can take on a number of interpretations. Define the $n \times n$ square matrix $A = [\alpha_{ij}]$. In one interpretation, α_{ij} measures the share of firm j owned by the manager of firm i, and each firm manager (or proprietor) operates its business in order to maximize her own simple total return. Alternatively, firm i management may recognize that profits it receives from firm j depend on firm j's partial ownership of firm k and so on. For this interpretation, total profits $U = (U_1, \ldots, U_n)$ are described in matrix form as $U = \Pi + DU$ where Π is the vector of operational profits and D describes cross ownership in the following manner: $d_{ii} = 0$, for all i and for all $i \neq j$, d_{ij} is the share interest in firm j owned by firm i.[19] Solving for U, one obtains $U = (I - D)^{-1}\Pi$. For this interpretation, $A = (I - D)^{-1}$.[20] While allowing for this possibility, for simplicity we refer to the elements of A as ownership shares.

Proof of Proposition 1. Clearly a bidder with a value below r is better off not bidding than submitting a bid greater than or equal to r. It is straightforward to check that when $v_i > r$, not bidding or above $b^F(\overline{v})$ by bidder i never results in a higher expected payoff than the proposed equilibrium bid. It remains to check that a bidder is not better off bidding as if he had a different value when all of the other bidders follow the proposed equilibrium strategy. This follows from the fact that $\partial u_i(v_i, b^F(v_i))/\partial\beta = 0$ for all $v_i \in (r, \overline{v}]$ and that $\partial u_i(v_i, b^F(v_i))/\partial\beta$ is increasing in its first argument. Making use of the following expression it is a straightforward exercise to check these conditions.

$$\partial u_i^F(v_i, \beta)/\partial\beta = (\alpha_{ii} - \bar{\alpha}_i)\left[v_i - \beta\right](n-1)F(\phi^F(\beta))^{n-2} f(\phi^F(\beta))\phi^{F\prime}(\beta) - \alpha_{ii}F(\phi^F(\beta)).$$

Q.E.D.

Proof of Proposition 2. In the two bidder case, the payoff to the bidder i in an English auction when bidder j bids according to b^E is

$$u_i^E(v_i, \beta) = \alpha_{ii}\int_r^{\phi^E(\beta)} \left[v_i - b^E(\xi)\right] f(\xi)d\xi + \alpha_{ij}\int_{\max\{\phi^E(\beta), r\}}^{\overline{v}} [\xi - \max\{\beta, r\}] f(\xi)d\xi,$$

[19] Flath (1991) uses this formulation, and Ettinger (2002, 2003) discusses it for the case of cross-ownership between two bidders.

[20] This formulation raises the question of the invertability of $I - D$ and the relationship between the ownership shares and the final weights. Mathematically the structure is identical to the input/output models estimated by Leontief (1951). Takayama (1985) presents an extensive treatment of the properties of $I - D$ sufficient for the existence and nonnegativity of the inverse.

where the function ϕ^E is the inverse of b^E. Also note that for $\beta > r$,

$$\begin{aligned}\partial u_i^E(v_i,\beta)/\partial\beta &= \alpha_{ii}\left[v_i - b^E(\phi^E(\beta))\right] f(\phi^E(\beta))\phi^{E\prime}(\beta) - \alpha_{ij}\left[\phi^E(\beta)-\beta\right] f(\phi^E(\beta))\phi^{E\prime}(\beta) \\ &\quad -\alpha_{ij}\left[1-F(\phi^E(\beta))\right]\end{aligned}$$

The proof is completed by the same arguments made in the proof of Proposition 1. *Q.E.D.*

Proof of Lemma 1. Let $\widetilde{u}_i^{\mathrm{X}}(v_i, v_i')$ and $\widetilde{u}_i^{\mathrm{Y}}(v_i, v_i')$ denote bidder i's expected payoff in auctions X and Y when he has value v_i but bids as if he had value v_i' (*i.e.*, in a first-price auction that would mean submitting the bid $b^F(v_i')$). For any auction X where the highest valued bidder always wins,

$$\widetilde{u}_i^{\mathrm{X}}(v_i, v_i') = \alpha_{ii}[v_i - p^{\mathrm{X}}(v_i')]F(v_i')^{n-1} \cdot 1_{\{v_i' \geqslant r\}} + \overline{\alpha}_i \psi_i^{\mathrm{X}}(v_i')$$

where the first term is bidder i's share of his expected payoff from winning times the probability of winning and $\psi_i^{\mathrm{X}}(v_i')$ is the expected payoff from winning of i's highest valued rival conditional on having a value above v_i'. Equilibrium behavior requires that for all $v_i, v_i' \in [r, \overline{v}]$,

$$\begin{aligned} 0 &\leqslant \widetilde{u}_i^{\mathrm{X}}(v_i, v_i) - \widetilde{u}_i^{\mathrm{X}}(v_i, v_i') \\ &= \widetilde{u}_i^{\mathrm{X}}(v_i, v_i) - \widetilde{u}_i^{\mathrm{X}}(v_i', v_i') + \widetilde{u}_i^{\mathrm{X}}(v_i', v_i') - \widetilde{u}_i^{\mathrm{X}}(v_i, v_i') \\ &= \widetilde{u}_i^{\mathrm{X}}(v_i, v_i) - \widetilde{u}_i^{\mathrm{X}}(v_i', v_i') + \alpha_{ii}[v_i' - v_i]F(v_i')^{n-1}. \end{aligned}$$

Hence,

$$\alpha_{ii}F(v_i)^{n-1} \geqslant \frac{\widetilde{u}_i^{\mathrm{X}}(v_i, v_i) - \widetilde{u}_i^{\mathrm{X}}(v_i', v_i')}{v_i - v_i'} \geqslant \alpha_{ii}F(v_i')^{n-1}$$

Thus, $d\widetilde{u}_i^{\mathrm{X}}(v_i, v_i)/dv_i = \alpha_{ii}F(v_i)^{n-1}$ and $\widetilde{u}_i^{\mathrm{X}}(v_i, v_i) = \alpha_{ii}\int_r^{v_i} F(\xi)^{n-1}d\xi \cdot 1_{\{v_i \geqslant r\}} + \widetilde{u}_i^{\mathrm{X}}(\underline{v}, \underline{v})$. In the previous expression X can be replaced with Y. Hence,$\widetilde{u}_i^{\mathrm{X}}(v_i, v_i) \gtreqless \widetilde{u}_i^{\mathrm{Y}}(v_i, v_i)$ if and only if $\widetilde{u}_i^{\mathrm{X}}(\underline{v}, \underline{v}) \gtreqless \widetilde{u}_i^{\mathrm{Y}}(\underline{v}, \underline{v})$. The result follow because revenues for auctions X and Y are the expected highest value net of the sum of the expected payoffs of the bidders. *Q.E.D.*

Proof of Proposition 3. For the case where both auctions have the same reserve price, we complete the proof by induction. We first show that when $n = 2$, the seller prefers a first-price auction to an English auction when the reserve prices are the same. We then show that if the seller prefers a first-price to an English auction when there are $n-1$ bidders, then the seller also prefers a first-price auction when there are n bidders.

Given Lemma 1, to show the result for $n = 2$, we need only show that $u_i^E(\underline{v}, b^E(\underline{v})) >$

$u_i^F(\underline{v}, b^F(\underline{v}))$. This is relatively straightforward since

$$u_i^E(\underline{v}, b^E(\underline{v})) = \alpha_{ij} \int_r^{\overline{v}} [v_j - r] f(v_j) dv_j > \alpha_{ij} \int_r^{\overline{v}} \left[v_j - b^F(v_j)\right] f(v_j) dv_j = u_i^F(\underline{v}, b^F(\underline{v})).$$

Now we show that if the seller prefers first-price to English auctions when there are $n-1$ bidders, then she also prefers first-price auctions when there are n bidders. This step is completed by using Lemma 1 and showing that if the seller prefers first-price to English auctions when there are $n-1$ bidders, then the expected payoff to a bidder with value $\underline{v}$ is higher in an English auction than a first-price auction, when there are n bidders.

Define $\tilde{b}^F(\cdot; A)$ as the equilibrium bidding strategy in a first-price auction given symmetric ownership matrix A.[21] For $v_i < r$, let $\tilde{b}^F(v_i; A) = 0$. Define A_{-k} as the matrix A with the kth column and row deleted. Since the ownership shares comprising A are symmetric, $A_{-k} = A_{-h}$, for all $k, h \in N$. We assume that when there are $n-1$ bidders, the seller prefers a first-price auction to an English auction. Hence,

$$E[\tilde{b}^F(V_1^{n-1}; A_{-k})] > E[P_{A_{-k}}^E] \tag{2}$$

where V_1^{n-1} is a random variable equal to the highest of $n-1$ draws from F and $P_{A_{-k}}^E$ is a random variable equal to the expected price paid in an English auction with $n-1$ bidders and ownership profile A_{-k}. In an English auction with n bidders the payoff of a bidder with value $\underline{v}$ is $\alpha_{ij} E[(V_1^{n-1} \cdot 1_{\{V_1^{n-1} \geqslant r\}} - P_{A_{-k}}^E]$. In a first-price auction with n bidders with ownership shares A the payoff of a bidder with value $\underline{v}$ is $\alpha_{ij} E[V_1^{n-1} \cdot 1_{\{V_1^{n-1} \geqslant r\}} - \tilde{b}^F(V_1^{n-1}; A)]$. The desired inequality that a bidder with value $\underline{v}$ prefers an English auction to a first-price auction follows from

$$E[\tilde{b}^F(V_1^{n-1}; A)] > E[\tilde{b}^F(V_1^{n-1}; A_{-k})] > E[P_{A_{-k}}^E].$$

The first inequality follows from inspecting the expression for b^F in Proposition 1 and seeing that b^F is increasing in n. The second inequality follows from Inequality (2).

For the case of optimally set reserve prices, let $R^E(A, r)$ and $R^F(A, r)$ denote respectively the seller's expected revenue from having an English auction and from a first-price auction. We have established for symmetric A and $r < \overline{v}$ that $R^E(A, r) < R^F(A, r)$. Letting $r^E(A)$ and $r^F(A)$ denote the corresponding optimal reserve prices, it then follows that $R^E(A, r^E(A)) < R^F(A, r^E(A)) \leq R^F(A, r^F(A))$. Thus the seller also prefers first-price auctions when reserve prices are set optimally. *Q.E.D.*

[21] Recall that the definition of symmetric ownership shares is stronger than assuming that the matrix A is symmetric. The number of bidders is captured in the dimensionality of A.

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