Rev. Econ. Design 8, 465–477 (2004)
DOI: 10.1007/s10058-004-0115-5

Review of Economic Design

Mechanisms for dividing labor and sharing revenue in joint ventures

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Received: 21 March 2002 / Accepted: 26 February 2004

Abstract. Organizing the productive efforts of firms participating in a joint venture involves assigning firms to tasks according to abilities. A multidimensional incentive problem arises when abilities are private information. In any equilibrium, it is better to be a firm who is a specialist rather than a generalist. However, generalists can expect to receive a larger allocation of revenue. If at least one firm is decisive to the profitability of the joint venture (i.e., if it can make a credible cost announcement that implies the joint venture earns zero profit), then the joint venture will not be able to implement a profit maximizing or cost minimizing production plan.

JEL classification: D82, C78, L23

Key words: Mechanism design, joint venture, team production

1 Introduction

Production usually involves the efforts of more than one agent, each possibly performing a different task. The agents could be workers or divisions within a firm, partners in a partnership, firms participating in a joint venture, or even firms within a market. When each agent's costs are private information, a system of incentives needs to be designed that elicits that information if it is to be used to allocate productive tasks and determine total output.

There is a well known literature that considers the problem of providing agents with the correct incentives when each agent's contribution to the total output is

Any views expressed in this paper are not purported to reflect those of the United States Department of Justice. This paper benefited from the helpful comments of a referee and an associated editor.

unobservable by the principal.[1] In these models each agent's effort level is an input to the firm's aggregate production function. The problem faced by the principal is to make each agent's compensation contingent on the firm's total output in the right way in order to give each agent an incentive to optimally contribute to total output. However, these moral hazard models ignore the problem of optimally assigning workers to different tasks.

There are many joint production environments where observing a worker's contribution to output is less problematic than implied by the moral hazard models. The most common forms of joint production are those where each worker performs a well-defined productive task. An assembly line is such a form of joint production. The efficiency of these forms of joint production derives from a division of labor according to comparative advantage. Each worker develops a relative expertise or has a relative aptitude for the productive task assigned to him. For firms participating in a joint venture, this form of production appears particularly prevalent. Each firm produces a part of the final product or assembles parts already produced by other participating firms. However, if each firm's expertise or aptitude is not apparent to the venture as a whole, then in order to produce efficiently, firms must be provided with the incentive to self-select into the productive task suited to them.

In this paper I consider the problem faced by a group of firms participating in a joint venture. The group must decide on a production plan where a production plan dictates the quantity to produce and the assignment of productive tasks to the participating firms. The group must also decide how to allocate the joint venture's revenues. However, each firm's costs of performing the different tasks are private information. If more than one task could be assigned to a given firm, then that firm's relevant private information is a vector of more than one dimension. That is, a participating firm has a different cost for each of its potential tasks, each of which is private information. Therefore, the incentive mechanism utilized by the joint venture must elicit a truthful report of a *vector* of information rather than just a scalar value. This gives rise to a multidimensional incentive problem.

There is a large literature on incentive contracts (e.g., Baron and Myerson 1982; Guesnerie and Laffont 1984; Maskin and Riley 1984). The analysis presented here is most closely related to incentive problems that require a balanced budget (e.g., Cramton et al. 1987; Cramton and Palfrey 1990; Makowski and Mezzetti 1994: Myerson and Sattherthwaite 1983). Darrough and Stoughton (1989) consider the problem of joint ventures. In their model, different production plans only entail changes in joint output and not assignment of tasks. Sherstyuk (1999) describes the optimal mechanism for a firm assigning teams of workers to different tasks, but the model assumes that the information structure is such that each team's private information can be summarized by a scalar value.

The model presented here is an application of what has been called multidimensional mechanism design. While the literature on incentive contracts and mechanism design is large, only a small subset of those papers consider problems that involve private information that is multidimensional. Even so, there are many

[1] See Holmstrom (1982). Eswaran and Kotwal (1984), Rasmusen (1987), and Sjöström (1996) comment on and extend the Holmstrom moral hazard model. McAfee and McMillan (1991) extends the Holmstrom model by adding adverse selection.

applications that multidimensional mechanism design techniques can inform—only some of which have been examined. Armstrong (1996, 1999), McAfee and McMillan (1988), and Rochet and Choné (1998) consider pricing strategies by a multiproduct monopolist. Jehiel and Moldovanu (1996), Jehiel et al. (1996, 1999) consider the selling of a good with externalities. Armstrong (1993) derives the optimal regulation of multiproduct firms when costs are unknown. Waehrer (2003) applies multidimensional mechanism design techniques to the problem of siting hazardous facilities. The present model represents another application where scalar mechanisms fail to match the scope of the problem.

I derive implications of incentive compatibility on the revenue sharing rules and production plans. Incentive compatibility implies that a participating firm's expected payment must not increase for a proportional rise in its marginal cost of performing each of the tasks. Higher expected payments tend to be made to firms that are equally skilled at all tasks rather than having specialized skills. However, the expected payoff must be convex in a firm's vector of costs. Hence, specialists receive a higher expected payoff than generalists. I characterize the production plans and revenue sharing rules that are Bayesian incentive compatible, interim individually rational, and ex ante balanced. I show that the joint venture is not able to maximize its joint profit if one firm is decisive in the sense that it could have costs such that the probability of a positive level of profit by the joint venture is zero.

2 The model

A group of firms must decide on a production plan that determines who should perform what tasks and how much is to be produced. I assume that each task is autonomous in that the costs associated with performing one task do not affect another and that a firm's output from performing its task is observable. One can think of tasks as comprising of the production or assembly of the pieces or parts of the good produced by the joint venture.

Let $N = \{1, \ldots, n\}$ ($n > 1$) index the set of firms who are participating in the joint production process. A production plan is a rule for allocating tasks and determining the level of output to be produced by the joint venture. Let $M = \{1, \ldots, m\}$ index the set of potential task allocations. Hence, I assume that there are a finite number of ways that the production process can be shared among the n firms.

Let c denote an $m \times n$ matrix of constant marginal costs where the element c_{ji} is firm i's constant marginal cost when task allocation $j \in M$ is selected. I assume that costs are nonnegative. Note that M indexes allocations of tasks rather than the tasks themselves. For example, suppose there are three tasks and three firms participating in the joint venture. If all three firms could perform any of the three tasks, then there are six different ways of allocating tasks. Therefore, firm i will have the same marginal cost for any two different task allocations when those two allocations assign the firm the same task. Although defining cost as being associated with task allocations rather than the tasks themselves may seem awkward, it does not interfere with the interpretation of the results but does simplify the notation requirements later in the paper.

The model also allows for the number of participating firms to be larger than the number of tasks. Hence, the model can incorporate competition among firms for tasks and the associated share of the revenue. Suppose there are three firms and two tasks. In such a case, define “not being assigned a task” as performing a task that has a marginal cost of zero.

While $c_i = (c_{1i}, \ldots, c_{mi})$ is known to firm i, it is considered a random vector by the other firms. I assume that the firms have a common belief regarding the distribution of $(c_1, \ldots, c_n)$ where each firm's cost vector is believed to be independent of the other firms' cost vectors. I assume that for each $i \in N$, the support of c_i, denoted $\mathcal{C}_i \subseteq \mathbb{R}^m$, is compact and convex. $E[\cdot]$ denotes the expectation operator with respect to the distribution of $(c_1, \ldots, c_n)$. Note that these assumptions allow for elements of c_i to be correlated (even perfectly correlated) and to be degenerate. Let $\mathcal{C} = \times_{i \in N} \mathcal{C}_i, \mathcal{C}_{-i} = \times_{j \in N \setminus \{i\}} \mathcal{C}_j$, and c_{-i} denote all of the columns of c except the ith.

Appealing to the revelation principle, I only consider direct mechanisms. Any outcome that can be achieved as a Bayesian equilibrium in an indirect game can be achieved as an equilibrium in a direct mechanism where the Bayesian equilibrium involves each firm truthfully announcing its costs. The presentation and analysis are further simplified by only considering deterministic mechanisms.

Definition 1. A *joint production direct mechanism* is a triple of functions $\langle S, A, Q \rangle$, $S : \mathcal{C} \to \mathbb{R}^n$, $A : \mathcal{C} \to \{0, 1\}^m$, and $Q : \mathcal{C} \to \mathbb{R}_+$.

The function S is the revenue allocation rule. $S(c) = (S_1(c), \ldots, S_n(c))$ where firm i receives $S_i(c)$ in revenue when the matrix c is announced by the n firms. The allocation of tasks is described by the mechanism through the function A. $A(c) = (A_1(c), \ldots, A_m(c))$ where $A_j(c) = 1$ implies that task allocation j is selected when the firms announce costs c. Similarly, $Q(c)$ is the quantity of output by the joint venture when c is the vector the firms' announced costs. I only consider production plans where Q is bounded. I refer to $\langle A, Q \rangle$ as a *production plan* of the joint venture. A production plan is the rule that maps every possible cost report to a task allocation and output level. For example, one possible production plan is the profit maximizing plan where, based on announced costs, tasks are allocated to minimize costs and output is set so that marginal revenue equals marginal costs.

When c is announced by the firms and firm i has cost vector c_i, then firm i's total cost is $c_i \cdot A(c)Q(c)$ and the joint venture's total cost is $Q(c) \sum_{i \in N} c_i \cdot A(c)$. The profit of the joint venture when each firm truthfully announces its costs is

$$\Pi(c) = R(Q(c)) - Q(c) \sum_{i \in N} c_i \cdot A(c)$$

where $R : \mathbb{R}_+ \to \mathbb{R}_+$ is the revenue function faced by the joint venture. Clearly, at a minimum, the sum of the firms' ex ante expected payoffs from the joint venture must equal the joint venture's expected profit. This fact implies the following ex ante balanced budget requirement.

Definition 2. The mechanism $\langle S, A, Q\rangle$ is *ex ante balanced* (B) if and only if

$$\sum_{i\in N} E[S_i(c)] = E[R(Q(c))].$$

Firm i's interim expected payoff when it has cost vector c_i but announced cost vector c_i' and all of the other firms truthfully announce their costs is

$$\begin{aligned}\pi^i(c_i', c_i) &= E_{-i}\left[S_i(c_i', c_{-i}) - c_i \cdot A(c_i', c_{-i})Q(c_i', c_{-i})\right]\\ &= s^i(c_i') - c_i \cdot q^i(c_i'),\end{aligned}$$

where E_{-i} denotes the expectation with respect to all of the costs except i's, $q^i(c_i) = E_{-i}[A(c)Q(c)]$, and $s^i(c_i) = E_{-i}[S_i(c)]$. In equilibrium, when all firms truthfully announce their costs, firm i's interim expected payoff is $v^i(c_i) = \pi^i(c_i, c_i)$.

Definition 3. The mechanism $\langle S, A, Q\rangle$ is *Bayesian incentive compatible* (IC) if and only if for all $i \in N$ and $c_i, c_i' \in \mathcal{C}_i$, $v^i(c_i) \geqslant \pi^i(c_i', c_i)$.

Note that $v^i(c_i) \geqslant \pi^i(c_i', c_i)$ if and only if

$$v^i(c_i) - v^i(c_i') \geqslant -[c_i - c_i'] \cdot q^i(c_i'). \tag{1}$$

I also require that the joint production mechanism be interim individually rational. That is, each firm's expected payoff after observing its own costs but before learning the resulting outcome must be greater than zero.

Definition 4. The joint production mechanism $\langle S, A, Q\rangle$ is *interim individually rational* (IR) if and only if for all $i \in N$ and $c_i \in \mathcal{C}_i$, $v^i(c_i) \geqslant 0$.

Definition 5. A production plan $\langle A, Q\rangle$ is *implementable* if and only if there exists a revenue sharing rule S such that $\langle S, A, Q\rangle$ is Bayesian incentive compatible, interim individually rational, and ex ante balanced.

Prior to presenting a characterization of implementable production plans in Sect. 4, I present a number of interesting implications of Bayesian incentive compatibility in the following section.

3 Incentive compatibility and its implications

The requirement that a mechanism be Bayesian incentive compatible places a number of economically interesting restrictions on the joint production mechanism. I begin this section by presenting a characterization of Bayesian incentive compatible joint production plans. This result makes use the concept monotonicity.

Definition 6. The vector-valued function $-q^i$ is *monotone* if and only if

$$-q^i(c_i) \cdot (c_i' - c_i) - q^i(c_i') \cdot (c_i - c_i') \leqslant 0,$$

for any pair of cost vectors $c_i, c_i' \in \mathcal{C}_i$.

Lemma 1. *The joint production mechanism $\langle S, A, Q\rangle$ is Bayesian incentive compatible if and only if for all $i \in N$ and $c_i, c_i' \in \mathcal{C}_i$,*

$$v^i(c_i) - v^i(c_i') = -\int_0^1 q^i(\alpha c_i + (1-\alpha)c_i') \cdot (c_i - c_i')d\alpha \tag{2}$$

and $-q^i$ is monotone.

The proof of this lemma is standard. For example see Waehrer (2003).[2]

Intuitively, the first-order condition associated with $v^i(c_i) = \max_{c_i' \in \mathcal{C}_i}\{\pi^i(c_i', c_i)\}$ implies the differential equation $\nabla v^i = -q^i$ holds almost everywhere and, hence, the integral in (2). Therefore, in any Bayesian incentive compatible joint production mechanism, differences in a firm's interim expected payoff are completely determined by the production plan.

The following proposition presents a list of properties implied by incentive compatibility that have interesting economic implications for the joint production application.

Proposition 1. *If $\langle S, A, Q\rangle$ is Bayesian incentive compatible, then for each $i \in N$,*

(a) v^i *is convex and nonincreasing,*
(b) $s^i(c_i) \geqslant s^i(c_i')$ *for all* $c_i \in \mathcal{C}_i$ *and* $\alpha > 1$ *such that* $c_i' = \alpha c_i \in \mathcal{C}_i$,
(c) $s^i(c_i) \geqslant s^i(c_i')$ *for all* $c_i \in \{\xi \in \mathcal{C}_i | \, \xi = (\delta, \dots, \delta), \delta \in \mathbb{R}_+\}$ *and* $c_i' \in \mathcal{C}_i$ *such that* $\sum_{j \in M} q_j^i(c_i) \geqslant \sum_{j \in M} q_j^i(c_i')$,
(d) $\sum_{j \in M} q_j^i(c_i) \geqslant \sum_{j \in M} q_j^i(c_i')$ *for all* $c_i, c_i' \in \{\xi \in \mathcal{C}_i | \, \xi = (\delta, \dots, \delta), \delta \in \mathbb{R}_+\}$ *such that* $c_i < c_i'$.

Proof in the Appendix. Part (a) of Proposition 1 is a standard result that can be found in a number of other multidimensional mechanism design papers. (For example see, McAfee and McMillan 1988; Rochet 1987.) It implies that the lower contour sets of v^i are convex. Hence, extremes are preferred to averages. (This is the opposite of the "averages are preferred to extremes" description of well-behaved preferences with convex upper contour sets.) Consider Fig. 1. $\mathcal{C}_i \subset \mathbb{R}^2$ is represented by the elliptical shape in the diagram. Cost vectors c_i and c_i' both lie on the same iso-payoff line. That is, $v^i(c_i) = v^i(c_i')$. For any $c_i'' = \alpha c_i + (1-\alpha)c_i'$ such that $\alpha \in (0, 1)$, $v^i(c_i'') \leqslant v^i(c_i)$. Therefore, in terms of a firm's interim expected payoff there is a sense that it is better to be a specialist than a generalist.

According to part (b) of the proposition, incentive compatibility implies that a firm's expected allocation of revenue is nonincreasing for proportional increases in the firm's marginal costs. That is, along any line passing through the origin such as the line in Fig. 2 that passes through c_i', the firm's expected allocation of revenue, s^i, does not get larger as the firm's vector of costs moves away from the origin.

[2] Other similar characterization theorems can be found in Jehiel et al. (1999), Krishna and Perry (1997), McAfee and McMillan (1988), Rochet (1985), and Williams (1998). The assumption that C is convex is used here since this characterization makes use of "straight" line integrals. However, this assumption can easily be weakened to assume that C is a path-connected set by making use of "smooth path" line integrals.

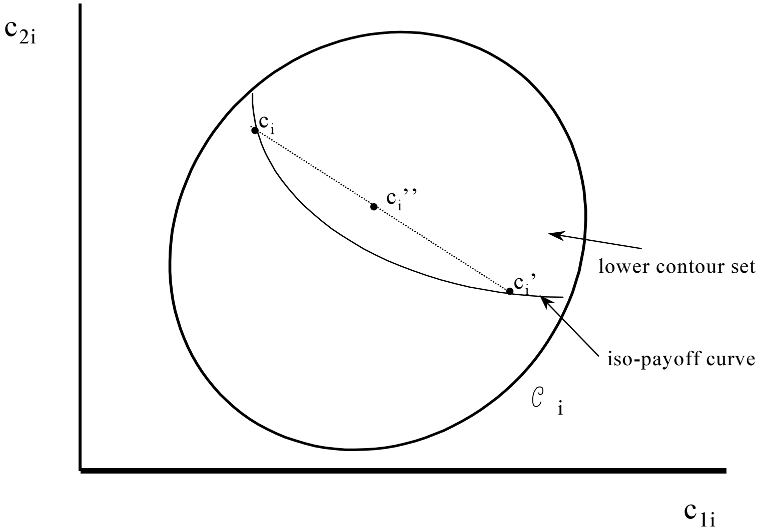

Fig. 1.

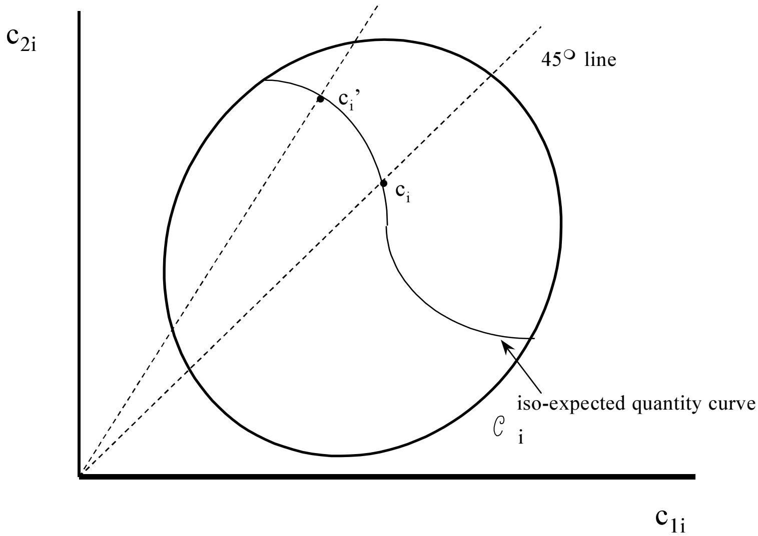

Fig. 2.

Part (c) also presents a comparative static result for the function s^i. Note that conditional on firm i's costs, the joint venture's expected output is $\sum_{j\in M} q_j^i(c_i)$ since $\sum_{j\in M} q_j^i(c_i) = E_{-i}[Q(c)\sum_{j\in M} A_j(c)]$. According to part (c), if $c_i \in \mathcal{C}_i$ is on the diagonal in $\mathbb{R}^m$, then $c_i \in \arg\max_{c_i'\in\mathcal{C}_i}\{s^i(c_i')|\sum_{j\in M} q_j^i(c_i') \leqslant \sum_{j\in M} q_j^i(c_i)\}$. Hence, compared to other cost vectors that generate the same or lower expected output, a vector of equal marginal cost for all task allocations is associated with the highest expected revenue allocation. In Fig. 2, the point c_i is associated with the highest expected allocation of revenue out of the cost vectors in $\mathcal{C}_i$ and to the northeast of the iso-expected quantity curve. Hence a generalist receives a larger expected allocation of revenue. This result follows from the fact that a firm that is indifferent between all task allocations has more bargaining power than a firm who cares which task it is assigned to. If a firm's cost vector is such that given a particular level of output the firm is indifferent between all task allocations, then the firm must be given the highest expected payment for announcing its true cost out of any other announcement that results in the same expected output.

4 Profit maximizing and cost minimizing production plans

Implementing a profit maximizing joint production plan involves satisfying Bayesian incentive compatibility and interim individual rationality while balancing payments to the participants with the revenues of the joint venture. In scalar mechanism design problems, the individual rationality constraint is reduced to a constraint on a single "worst type" by making use of the fact that incentive compatibility implies that v^i is nonincreasing. In this multidimensional problem the same technique can be used. However, identifying the "worst type" is more difficult since there is not necessarily one cost vector that can be identified as the highest. If the $\mathcal{C}_1, \ldots, \mathcal{C}_n$ are rectangular (as in Jehiel et al. 1999), then there is a unique cost level that is highest regardless of the production plan. (By making a weaker assumption regarding the shapes of $\mathcal{C}_1, \ldots, \mathcal{C}_n$ than their being rectangular, Armstrong (1996) also avoids any ambiguity regarding the worst type.) However, in the more general case where $\mathcal{C}_1, \ldots, \mathcal{C}_n$ are convex, there can be a large number of candidates for the "worst" type. For example, in Figs. 1 and 2, any point on the upper right hand boundary of $\mathcal{C}_i$ could be the worst type. As is evident from the analysis below, the "worst type" can depend on the production plan.

Define $\bar{c}_i \in \mathcal{C}_i$ such that for all $c_i \in \mathcal{C}_i, -\int_0^1 q^i(\alpha c_i+(1-\alpha)\bar{c}_i)\cdot(c_i-\bar{c}_i)d\alpha \geqslant 0$. Clearly, there is no $c_i \in \mathcal{C}_i$ such that $c_i > \bar{c}_i$ unless $q^i(\alpha c_i + (1-\alpha)\bar{c}_i) = 0$ for almost all $\alpha \in [0,1]$. When $-q^i$ is from an Bayesian incentive compatible joint production mechanism, $\bar{c}_i$ exists since finding $\bar{c}_i$ is equivalent to finding the minimum of a continuous function on a compact set.

Lemma 2. *If the mechanism $\langle S, A, Q\rangle$ is Bayesian incentive compatible and for all $i \in N$, $v^i(\bar{c}_i) \geqslant 0$, then the mechanism is interim individually rational.*

Proof. By incentive compatibility and the definition of $\bar{c}_i$,

$$(\forall i \in N, \forall c_i \in \mathcal{C}_i) \qquad v^i(c_i)-v^i(\bar{c}_i) = -\int_0^1 q^i(\alpha c_i+(1-\alpha)\bar{c}_i)\cdot(c_i-\bar{c}_i)d\alpha \geqslant 0.$$

Therefore, $v^i(\bar{c}_i) \geqslant 0$, for all $i \in N$ implies $v^i(c_i) \geqslant 0$, for all $i \in N$ and $c_i \in \mathcal{C}_i$ and the mechanism is interim individually rational. □

I now turn to the question of whether or not the joint venture can achieve the same level of profits that could be achieved in a complete information environment. A profit maximizing production plan must produce a given output at minimum cost and produce an output level so that the marginal revenue of the joint venture is equal to the marginal cost. Hence, the profit maximizing production plan must be able to allocate firms to the different tasks so as to minimize production costs while at the same time determine the optimal output level of the joint venture.

Definition 7. A production plan $\langle A, Q\rangle$ with joint profit Π is *profit maximizing* if and only if for any production plan $\langle \hat{A}, \hat{Q}\rangle$ with joint profit $\hat{\Pi}$, $\Pi(c) \geqslant \hat{\Pi}(c)$, for all $c \in \mathcal{C}$.[3]

A cost minimizing production plan simply must allocate tasks in a cost minimizing way without having to determine the optimal output level. A cost minimizing problem could arise in situations where a joint venture has contracted with a consumer such as a governmental body to supply a specified quantity of output in exchange for a fixed payment.

Definition 8. A production plan $\langle A, Q\rangle$ is *cost minimizing* if and only if Q is positive and constant and for any allocation of tasks $\hat{A}$, $\sum_{i\in N} c_i \cdot \left[A(c) - \hat{A}(c)\right] \leqslant 0.$

It is clearly the case that if $\langle A, Q\rangle$ is a profit maximizing production plan, then the task allocation rule A must be cost minimizing. That is, it must be the case that for any matrix of costs $c \in \mathcal{C}$ and for any other allocation rule $\hat{A}$, $\sum_{i\in N} c_i \cdot A(c) \leqslant \sum_{i\in N} c_i \cdot \hat{A}(c)$.

A profit maximizing joint production mechanism is equivalent to an efficient mechanism as it is defined in other mechanism problems (e.g., Cramton et al. 1987). Conditional on the constraint inherent in the revenue function R, a profit maximizing joint venture implies that all of the gains available to the joint venture are extracted. I avoid use of the term efficiency in the current context because in a broader sense a profit maximizing production plan may not be efficient since it is likely to involve the consumption distortions associated with monopoly pricing.

In the following lemma, I exploit the relationship between efficient mechanisms as defined in other applications and profit maximizing and cost minimizing production plans in the present context. The following lemma is standard (e.g., D'Aspremont and Gérard-Varet 1979; Krishna and Perry 1997; Williams 1999).

Lemma 3. *Suppose the production plan $\langle A, Q\rangle$ is either profit maximizing or cost minimizing. Then the mechanism $\langle S, A, Q\rangle$ is Bayesian incentive compatible if and only if*

$$(\forall i \in N, \forall c_i, c_i' \in \mathcal{C}_i) \qquad v^i(c_i) - v^i(c_i') = E_{-i}\left[\Pi(c_i, c_{-i}) - \Pi(c_i', c_{-i})\right]. \quad (3)$$

This lemma has been described as implying that every profit maximizing (i.e., efficient) Bayesian incentive compatible mechanism must be a Groves mechanism in expectation (see Makowski and Mezzetti 1994). In a Grove mechanism each agent is given the total gains from trade plus or minus a "trading charge" that is

[3] The existence of a profit maximizing production plan is not guaranteed by the assumptions made thus far. However, it is sufficient to make the standard assumptions with regard to the revenue function. That is, the revenue function is derived from a continuous and downward sloping demand curve.

constant in the firm's own cost vector. This property makes truth-telling a dominant strategy when the mechanism maximizes the gains from trade. For Bayesian incentive compatibility, it is necessary and sufficient to give each agent the expected gains from trade plus or minus a trading charge.

Note that Lemma 3 implies that for any profit maximizing or cost minimizing production plan, $-q^i$ is monotone since there exists an Bayesian incentive compatible mechanism for production plans. Using Lemma 3 the following characterization is possible.[4]

Theorem 1. *A profit maximizing or cost minimizing production plan $\langle A, Q\rangle$ is implementable if and only if*

$$E\left[\sum_{i\in N} \Pi(\bar{c}_i, c_{-i}) - (n-1)\Pi(c)\right] \geqslant 0. \tag{4}$$

It is not a trivial matter to satisfy (4) since for profit maximizing production plans, (3) and interim individual rationality imply $E\left[\Pi(c) - \Pi(\bar{c}_i, c_{-i})\right] \geqslant 0$, with a strict inequality in most cases. Hence, to satisfy (4), $\sum_{i\in N} E\left[\Pi(c) - \Pi(\bar{c}_i, c_{-i})\right]$ must be smaller than $E\left[\Pi(c)\right]$. The condition is more likely to be satisfied the larger $E\left[\Pi(\bar{c}_i, c_{-i})\right]$ is relative to $E\left[\Pi(c)\right]$.

Given sufficient revenues that do not vary with cost levels, it is possible to satisfy (4). Consider an example where production is always profitable, but only one unit is demanded. Hence, $Q(c) = 1$, for all $c \in \mathcal{C}$, and costs must be finite. Define $P = R(1)$ Under this assumption, inequality (4) can be rewritten as

$$P - E\left[\sum_{i\in N}\left(\bar{c}_i + \sum_{j\in N\setminus\{i\}} c_j\right)\cdot A(\bar{c}_i, c_{-i}) - (n-1)\sum_{i\in N} c_i \cdot A(c)\right] \geqslant 0.$$

The inequality above will be satisfied for sufficiently large P since costs are finite.

However, as demonstrated by the following proposition, profit maximization is not possible no matter how many firms participate, if there is at least one firm that is decisive to the profitability of the joint venture. Firm k is *decisive to the joint venture's profitability for production plan* $\langle A, Q\rangle$ if there is a $\tilde{c}_k \in \mathcal{C}_k$ such that under that production plan $E_{-k}[\Pi(\tilde{c}_k, c_{-k})] = 0$.

Proposition 2. *For a profit maximizing or a cost minimizing production plan $\langle A, Q\rangle$, if $E\left[\Pi(c)\right] > 0$ and there is at least one firm that is decisive to the joint venture's profitability, then $\langle A, Q\rangle$ cannot be implemented.*

Proof. I complete the proof by showing that $E_{-k}\left[\Pi(\bar{c}_k, c_{-k})\right] = 0$ and incentive compatibility imply that (4) cannot hold. Suppose firm k is decisive. By definition, there exists a $\tilde{c}_k \in \mathcal{C}_k$ such that $E_{-k}[\Pi(\tilde{c}_k, c_{-k})] = 0$. It follows that

$$0 \leqslant v^k(\tilde{c}_k) - v^k(\bar{c}_k) = E_{-k}\left[\Pi(\tilde{c}_k, c_{-k}) - \Pi(\bar{c}_k, c_{-k})\right] = -E_{-k}\left[\Pi(\bar{c}_k, c_{-k})\right].$$

[4] Williams (1999) shows this result. Makowski and Mezzetti (1994) provides the same result assuming that budgets must be balanced ex post.

The first inequality in the expression above follows from (IC) and the definition of $\bar{c}_k$. The first equality follows from (IC) assuming profit maximization or cost minimization. The second equality follows from $E_{-k}[\Pi(\tilde{c}_k, c_{-k})] = 0$. Therefore, $E_{-k}\left[\Pi(\bar{c}_k, c_{-k})\right] \leqslant 0$,[5] and

$$
\begin{aligned}
E\left[\sum_{i\in N}\Pi(\bar{c}_i, c_{-i}) - (n-1)\Pi(c)\right] &\leqslant \sum_{i\in N\setminus\{k\}} E\left[\Pi(\bar{c}_i, c_{-i}) - \Pi(c)\right] \\
&\quad \text{(since } E[\Pi(\bar{c}_k, c_{-k})] \leqslant 0\text{)} \\
&= \sum_{i\in N\setminus\{k\}} E\left[E_{-i}\left[\Pi(\bar{c}_i, c_{-i}) - \Pi(c)\right]\right] \\
&= \sum_{i\in N\setminus\{k\}} E\left[v^i(\bar{c}_i) - v^i(c_i)\right] < 0 \\
&\quad \text{(by (IC) and Lemma 3).}
\end{aligned}
$$

The final strict inequality follows from the fact that interim individual rationality implies for all $i \in N$, $E[v^i(c_i)] \geqslant v^i(\bar{c}_i)$ and for at least one $i \in N\setminus\{k\}$, $E[v^i(c_i)] > v^i(\bar{c}_i)$. To see that $E[v^i(c_i)] > v^i(\bar{c}_i)$ for at least one $i \in N$, note that $E\left[\Pi(c)\right] > 0$ implies that $Q(c) > 0$ for a nonnegligible subset of $\mathcal{C}$ and, hence, that $E_{-i}[Q(c_i, c_{-i})A(c_i, c_{-i})] = q^i(c_i) > 0$ for a nonnegligible subset of $\mathcal{C}_i$.[6] Therefore, v^i is nonconstant for a nonnegligible subset of $\mathcal{C}_i$, and thus, $E[v^i(c_i)] > \min_{c_i\in\mathcal{C}_i}\{v^i(c_i)\} = v^i(\bar{c}_i)$. □

Suppose that all n firms are needed to perform a task in order to produce any output. It could easily be possible for one firm's costs to be high enough so as to make profitable production impossible. If high cost firms can be excluded from the production process, then even if one firm has very high costs it may be possible for the joint venture to be profitable. Therefore, a firm is unlikely to be decisive in cases where its production tasks could be cost effectively performed by another firm. As demonstrated above, if the value of the joint venture's output is sufficiently valuable, then the profit maximizing production plan can be implemented. Proposition 2 implies that it is necessary for the implementation of the profit maximizing production plan that the joint venture's output must valuable enough to ensure that no firm is decisive.

Proposition 2 implies the impossibility of profit maximizing or cost minimizing joint production is the presence of a decisive firm. However, the result certainly does not imply that no production is possible, just that an optimal production plan cannot be implemented. The result can be likened to models where a certain firm controls a bottleneck in production and exploits that bottleneck to extract rents that distort markets away from efficient outcomes. However, simply being the only firm able to perform a critical task is not enough to create an efficiency problem. The problem arises if it is in the realm of possiblity that the firm might have costs so high as to make profitable production impossible.

[5] If the production plan is profit maximizing, then profits must be nonnegative. However, under cost minimizing production plans, profits can be negative.

[6] By $q^i(c_i) > 0$, I mean that all of the elements of the vector are nonnegative and at least one is strictly positive.

5 Discussion

The model describes a multilateral bargaining game within the context of a joint venture. However, one can also think of the model describing a market of firms such as electricity generating plants. In such a case the market determines which plants should produce at what times (i.e., the problem of allocating tasks) and the revenue earned by each plant (i.e., the revenue sharing rule). In fact any multi-firm market can be thought of as a joint production problem. In a decentralized market environment the requirement that the mechanisms be balanced simply implies that the total revenue in the market must equal the sum of the revenues of the individual firms. However, there may be additional restrictions on how revenues are split in decentralized market mechanisms. For instance, side-payments between firms may be restricted by antitrust rules in decentralized market mechanisms while contracts that involve side-payments are certainly allowed in joint ventures.

Appendix

Proof of Proposition 1. Part (a): The proof is standard. For example, see Rochet (1987).

Part (b): Incentive compatibility implies that $-q^i$ is monotone. Thus,

$$-q^i(c_i) \cdot (\alpha c_i - c_i) - q^i(\alpha c_i) \cdot (c_i - \alpha c_i) \leqslant 0.$$

For $c_i' = \alpha c_i$, rearranging this expression yields $(1-\alpha)[q^i(c_i) - q^i(c_i')] \cdot c_i \leqslant 0$. Therefore,

$$s^i(c_i) - s^i(c_i') \geqslant [q^i(c_i) - q^i(c_i')] \cdot c_i \geqslant 0,$$

where the first inequality follows from incentive compatibility.

Part (c): Suppose that $c_i = (\delta, \dots, \delta)$. Therefore,

$$\begin{aligned} s^i(c_i) - s^i(c_i') &\geqslant [q^i(c_i) - q^i(c_i')] \cdot c_i \\ &= \delta \left[\sum\nolimits_{j \in M} q^i_j(c_i) - \sum\nolimits_{j \in M} q^i_j(c_i') \right] \geqslant 0. \end{aligned}$$

The first inequality follows from incentive compatibility, and the final inequality follows the fact that $\sum_{j \in M} q^i_j(c_i) \geqslant \sum_{j \in M} q^i_j(c_i')$.

Part (d): The monotonicity of $-q^i$ follows from incentive compatibility by Lemma 2. Hence,

$$[q^i(c_i) - q^i(c_i')] \cdot (c_i - c_i') \leqslant 0.$$

Suppose that $c_i = (\delta, \dots, \delta)$ and $c_i' = (\delta', \dots, \delta')$. Therefore, $[\sum_{j \in M} q^i_j(c_i) - \sum_{j \in M} q^i_j(c_i')](\delta - \delta') \leqslant 0$. The conclusion follows from the fact that $\delta < \delta'$. □

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