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The ratchet effect and bargaining power in a two-stage model of competitive bidding*

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Summary. In the model presented, a buyer uses competitive bidding to facilitate her purchase of a good (the *primary* good of the exchange). Not included in the original purchase is the possible procurement of a good related to the original purchase: the *supplementary* good. The primary and supplementary goods are closely related; knowing a bidder's cost of producing the primary good implies that the buyer can infer the bidder's cost of producing the supplementary good. I show that a bidding mechanism for the primary good will fail to ensure an efficient allocation if the buyer learns the bid of the winner and the price of the supplementary good is determined through sequential bargaining.

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1 Introduction

Competitive bidding is widely used to facilitate exchange in the presence of asymmetric information. Suppose that an economic agent wishes to purchase a good. Before an exchange can take place, a price and a supplier must be chosen. When the buyer is relatively uninformed about the costs of the potential suppliers, competitive bidding is believed to be a good way to select a supplier and set the price. In fact, most of the models of competitive bidding found in the literature predict that the winning bidder is the supplier with the lowest cost. However, the models which exhibit this result for the most part analyze competitive bidding as a one time interaction of the buyer and

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sellers. In this paper I model an auction as the beginning of a long-term commercial relationship between the buyer and seller that may include other transactions not governed by the auction.

Specifically, a buyer uses competitive bidding to facilitate her purchase of a good (the *primary* good of the transaction). Not included in the original purchase is the possible procurement of a good related to the original purchase: the *supplementary* good. For example, the supplementary good could be spare parts for the primary good. Whatever the supplementary good, the private information of each bidder regarding his production cost of the primary good must also be informative about his production cost of the supplementary good. That is, if the buyer knows the winning bidder's cost for the primary good, then he can infer the bidder's cost for the supplementary good. With some probability the buyer decides to purchase the supplementary good from the winning bidder. The buyer and seller then bargain over its price. The bargaining follows a form of sequential bargaining found in Admati and Perry (1987) and Cramton (1992).

A natural application for this model is in the area of defense procurement. Once a supplier of a particular military aircraft is selected, any subsequent orders for additional aircraft will be made with the original supplier. If the price of subsequent orders are not specified by the original contract, then it is likely that the price for the additional aircraft will be arrived at through some sort of bargaining process.

Most auction models in the literature treat competitive bidding as completely isolated from any economic context.[1] Treating auctions in isolation is justified when bidding regulates a one-time short-term exchange. However, when competitive bidding is over a long-term contract, the results from these analyses may be insufficient. A few researchers have analyzed competitive bidding for long-term contracts. McAfee and McMillan (1986 and 1987b) examine competitive bidding for incentive contracts. Riley (1988) shows that the use of ex post information in long-term relationships increases the revenue of the auctioneer. Spulber (1990) and Waehrer (1995) analyze the effects of imperfect contract enforcement on competitive bidding. Haile (1996) considers bidding over goods with a resale market. Rothkopf, Teisberg, and Kahn (1990) and Engelbrecht-Wiggans and Kahn (1991) discuss how truth telling may not be an equilibrium in second-price auctions when the revealed information might be used against the winning bidder in later transactions.

In some of these models, the bid-taker's updated beliefs about the winning bidder's private information after observing the winning bid plays no part in the game. In McAfee and McMillan (1986 and 1987b) and Riley (1988), the auctioneer commits to an outcome in the period that follows the bidding. In Spulber (1990) and Waehrer (1995), either the bid-taker costlessly observes the private information of the winning bidder or has too little bargaining power in the second stage to use the information inferred in the first stage.

[1] For a review of the auction literature see McAfee and McMillan (1987a), Milgrom (1987), and Wilson (1992).

In Rothkopf, Teisberg, and Kahn (1990) and Engelbrecht-Wiggans and Kahn (1991) the bid-taker's updated beliefs about the winning bidder's private information after observing the winning bid does effect the play of later stages of the game; they present a similar nonexistence result as is presented here. In Haile (1996) the winning bidder's revealed information does effect play in later stages, but nonexistence of separating equilibria is avoided by assuming that the winning bidder's expected payoff in the second-stage is differentiable in the revealed information.

When the seller has the ability to make a take-it-or-leave-it offer in the bargaining over the supplementary good, the ratchet effect (Laffont and Tirole (1988)) implies that perfect information revelation in the first stage is impossible. If bidders follow a separating bidding strategy in the first stage of the game, then the seller would be able to infer the winning bidder's cost for the secondary good and make a take-it-or-leave-it offer that gives the winning bidder a payoff of zero in the second stage. Under these conditions, the bidders have an incentive to defect from the separating bidding strategy by "over bidding" so as to make the buyer believe they have higher costs than they actually have.

This paper demonstrates that perfect information revelation in the first stage may be impossible even when the buyer does not have the ability to make a take-it-or-leave-it offer. When the price in the second stage is determined by sequential bargaining, perfect information revelation in the first stage is impossible. The proof of the result relies on a number of strong assumptions. For instance, I assume that the expected cost of the supplementary good can be inferred from the seller's bid on the primary good. However, since an equivalent assumption is made by Laffont and Tirole (1988), the result does serve to show that the ratchet effect extends to environments where bargaining power is equally distributed in the second stage.

2 The model

A buyer decides to use a competitive bidding mechanism to determine a supplier for a good. Each bidder has private information about his cost of producing the good. The price and the supplier are determined through competitive bidding. Sometime after the bidding, the buyer decides whether or not to purchase a supplementary good from the winning bidder. I assume that the nature of the supplementary good is such that it is infeasible for any of the nonwinning bidders to produce it. The price of the supplementary good is determined through sequential bargaining between the winning bidder and the buyer.

The bidders are indexed by the set $N = \{1, \dots, n\}$. The cost of production for the representative bidder i is determined by his private information x_i. The ith bidder's cost of producing the primary good is denoted $c_P(x_i)$ where c_P is a decreasing function common to all bidders. The ith bidder's cost of producing the supplementary good is denoted $c_S(x_i)$ where c_S is also a decreasing function common to all bidders. Here higher private information values are considered good news, so the cost functions c_P and c_S are decreasing in x_i.

Let the private information of every bidder be independently distributed with distribution function Ψ. Assume that Ψ is continuous and strictly increasing on $[\underline{x}, \bar{x}] = \mathscr{X}$. Let $G(x_i)$ denote the probability that the bidders $N \backslash \{i\}$ have private information values less than x_i. Since the private information of the bidders is i.i.d., G can be written as $G(x_i) = \Psi(x_i)^{(n-1)}$. The supplementary good is purchased with probability λ. The probability that the supplementary good purchased is exogenous and independent of other events defined in the model.

Bids are submitted for the supply of the primary good. The bidder with the lowest bid is selected as the winner. The winning bidder receives a price equal to his bid and supplies the primary good at that price. Hence, the bidding scheme is a first-price auction within the context of a procurement problem.

The bidding strategy of supplier i is a function of the bidder's private information, denoted β_i. A bidding strategy is said to be *separating* if it is a strictly decreasing function of a bidder's private information. If the lowest cost bidder wins the bidding with probability one, then bidding strategies must be separating and symmetric almost everywhere. Suppose that a bid is submitted using a separating bidding strategy. Then if the bid is observed by someone knowing the strategy used by the bidder, he would be able to infer the private information of the bidder. Therefore, if the buyer is able to observe all of the bids, she would be able to infer the private information of every bidder.

The inference of the winning bidder's private information from his bid plays an important part in bargaining over the price of the supplementary good. If the winning bid is b and the equilibrium strategy used by the winning bidder is β_i, then the buyer would believe the seller to have private information $\beta_i^{-1}(b)$. Based on the winning bid b, the buyer believes that $c_S(\beta_i^{-1}(b))$ is the production cost for the supplementary good. If the seller bids according to the equilibrium strategy, then $b = \beta_i(x_i)$ and the buyer's beliefs about the seller's cost are correct.

3 Bargaining

The purpose of this section is to describe the outcome of sequential bargaining when the buyer believes that the bidders are following a symmetric and separating bidding strategy. Thus, the analysis of the sequential bargaining presupposes the revelation of the winning bidder's private information by the buyer prior to the start of bargaining.

The outcome of sequential bargaining depends on the buyer's beliefs about the winning bidder's cost of producing the supplementary good. These beliefs depend on the information she can infer from the bid of the winning bidder. As discussed above, if a decreasing bidding strategy is followed by the winning bidder, then the buyer's inference is correct. However, if the winning bidder defects from the decreasing bidding strategy, then the buyer's inference is incorrect.

The sequential bargaining process used here is very similar to the models of Admati and Perry (1987) and Cramton (1992). At the bargaining stage the buyer and seller alternate offers. Assume that the buyer makes the first offer. If the seller rejects the buyer's offer, then the seller either terminates negotiation or makes a counter offer after some delay Δ. The counter offer cannot be made sooner than z after the initial offer. Thus, $z > 0$ is the minimum time between offers.

Let v denote the value of the good to the buyer, and let c denote the cost of the good to the seller; v is common knowledge to buyer and seller; and c is the private information of the seller. Recall $\mathscr{X} = [\underline{x}, \bar{x}]$. It is common knowledge that $c \in \mathscr{C} = [\underline{c}, \bar{c}]$, where $\mathscr{C} = [c_S(\bar{x}), c_S(\underline{x})]$. If F_0 denotes the buyer's beliefs regarding the seller's cost after observing the bids but before the bargaining, then the support of the buyer's beliefs about the value c is always a subset of $\mathscr{C}$, (i.e., supp $F_0 \subseteq \mathscr{C}$).

An outcome of the bargaining stage is characterized by the resulting price p and the delay t before an offer is accepted. If no trade occurs, then $t = \infty$ and $p = 0$. I assume that the time preference of the bargainers can be described by a common discount rate $r > 0$ such that the payoff to the buyer from outcome $\langle t, p \rangle$ is $(v - p)e^{-rt}$ and the payoff to the seller from $\langle t, p \rangle$ is $\phi(c, t, p) = (p - c)e^{-rt}$. When time preferences are represented as above by a discount rate, then preferences are stationary and satisfy the single crossing property. The following lemma is a result of the single crossing property.

Lemma 1. *Suppose that for some* c, p, p', $t > 0$, *and* $\phi(c, 0, p) = \phi(c, t, p')$. *Then*
(i) *for all* $c' > c$, $\phi(c', 0, p) < \phi(c', t, p')$,
(ii) *for all* $c' < c$, $\phi(c', 0, p) > \phi(c', t, p')$.

Proof. The conclusion of the lemma follows from the observation that the difference $\phi(c, t, p') - \phi(c, 0, p) = e^{-rt}p' - p + (1 - e^{-rt})c$ is increasing in c. QED

Let $j = 1, 2, \ldots$ index the offers where $j = 1$ indexes the initial offer made by the buyer. There is a price and a period of delay associated with each offer. The initial offer is assumed to be made after no delay. Let p_j denote the jth offer made after a delay of Δ_j. As in Admati and Perry (1987) and Cramton (1992), I assume that the players determine the time between offers, $\Delta_j \geq z$. Thus, the jth offer is characterized by the pair (Δ_j, p_j). A history after offer j is a record of the winning bid and bidder (b, i) and all offers up to and including the jth. For example, $h^j = \{(b, i), (\Delta_1, p_1), \ldots, (\Delta_j, p_j)\}$. ($h^0 = \{(b, i)\}$.) Let H^j denote the set of all feasible histories following offer j. Suppose that the kth offer is accepted. Then the outcome of the bargaining stage is characterized by the accepted price $p = p_k$ and the time elapsed before the offer k was accepted $t = \sum_{j=1}^{k} \Delta_j$.

Let η_B and η_S denote a pure bargaining strategies for the buyer and seller respectively, (i.e., strategies for the bargaining stage of the game). When a player is called upon to take an action, he must either accept the previous offer or make a counter offer. (Termination of negotiations corresponds to an offer with infinite delay.) A strategy for the buyer, η_B, maps every possible history following an offer by the seller into the action space. (i.e., $\eta_B : H_B \to [z, \infty) \times \Re_+$, where $H_B = H^0 \cup H^2 \cup H^4 \cup \ldots$) A strategy for the seller, η_S, maps the product of $\mathscr{C}$ and every possible history following an offer by the buyer into the action space. (i.e., $\eta_S : \mathscr{C} \times H_S \to [z, \infty) \times \Re_+$, where $H_S = H^1 \cup H^3 \cup \ldots$) Let $\eta = (\eta_B, \eta_S)$.

Let $\mathscr{F}$ denote a system of beliefs for the buyer. $\mathscr{F} = F(\cdot|\cdot)$ where $F(\cdot|h^j)$ denotes the distribution function corresponding to the buyer's beliefs after h^j. To simplify the notation, let $F_j = F(\cdot|h^j)$. Clearly, if the buyer makes the j th offer, then $F_{j-1} = F_j$ (i.e., the buyer does not update her beliefs after her own offer).

While other equilibrium refinements have been applied to sequential bargaining under incomplete information, the most commonly used equilibrium concept in this literature is that of sequential equilibrium originally due to Kreps and Wilson (1982). "A *sequential equilibrium* for a game is a pair $(\eta, \mathscr{F})$ of strategies and beliefs, such that after every history h^j each player's strategy is optimal given the other's strategy and his current beliefs about the other's valuation, and his beliefs are consistent with Bayes' rule" (Cramton (1992)).

Suppose the bidder i wins the bidding with bid b. Given that he followed a decreasing strategy β_i, bidder i bids b, in equilibrium, only if he has private information $\beta_i^{-1}(b)$. Thus, Bayes' rule requires that $F_0 = \Phi_{\hat{c}}$ where $\hat{c} = c_S(\beta_i^{-1}(b))$ and $\Phi_{\hat{c}}$ denotes the degenerate distribution on $\hat{c}$.

If an offer is made by the seller in equilibrium with positive probability, then Bayes' rule determines how the buyer updates her beliefs. However, if an out-of-equilibrium offer is made, then in the absence of equilibrium refinements beyond sequential rationality, there are no restrictions on the buyer's updated beliefs. The lack of restrictions on the buyer's beliefs at unreached information sets allow many equilibria to be supported in standard models of bargaining under incomplete information. Along these lines Ausubel and Deneckere (1989) prove a folk theorem where out-of-equilibrium beliefs are the threats that are used to support a continuum of sequential equilibria. If the seller defects from the equilibrium, then the buyer punishes the seller by adopting beliefs that are least advantageous to the seller.

These equilibria satisfy the requirements of sequential rationality because there are no credibility requirements on out-of-equilibrium beliefs. Some researchers have reduced the set of possible equilibria by imposing some restrictions on the formation of beliefs in unreached information sets. For bargaining under incomplete information Rubinstein (1985) proposes a number of such restrictions on the adoption of beliefs when out-of-equilibrium information sets are reached. The following restriction generalizes two of the restrictions proposed by Rubinstein (1985).

Nonspurious Conjectures (NS) Suppose the buyer's offer (Δ_{j-1}, p_{j-1}) is rejected by the seller, and the seller makes the out-of-equilibrium offer (Δ_j, p_j). Then F_j is said to be a *nonspurious conjecture* if the following is satisfied.

Define the set k_j, a subset of $\mathscr{C}$, as the set of seller cost types who expect to receive a higher payoff by making the offer (Δ_j, p_j) when the buyer updates her beliefs to F_j than by accepting the offer (Δ_{j-1}, p_{j-1}).

(a) If $k_j \cap \operatorname{supp} F_{j-1} \neq \emptyset$, then $\operatorname{supp} F_j \subseteq \operatorname{supp} F_{j-1}$.
(b) If $k_j \cap \operatorname{supp} F_{j-1} = \emptyset$, then $\operatorname{supp} F_j \subseteq \operatorname{supp} F_{j-1} \cup k_j$.

Any system of beliefs that satisfies passive conjectures or rationalizing conjectures (Rubinstein (1985)) also satisfies nonspurious conjectures. For beliefs to be consistent with nonspurious conjectures, if a cost type is included in the support of the buyer's beliefs when it had previously not been in the support of the buyer's beliefs, then it must be the case that the types in the support of the buyer's previous beliefs would be worse off by making the out-of-equilibrium offer and all of the new types included in the support of the buyer's beliefs would be better off having made the out-of-equilibrium offer.

Lemma 2. *If updated beliefs satisfy* (*NS*), *then for any beliefs F_{j-1} and F_j,*

$$\inf\{\operatorname{supp} F_{j-1}\} \leq \inf\{\operatorname{supp} F_j\} \ .$$

Proof. If the jth offer is not out-of-equilibrium, then beliefs are updated using Bayes' rule. Thus, $\operatorname{supp} F_j \subseteq \operatorname{supp} F_{j-1}$, and the conclusion of the lemma follows.

Suppose that the jth offer is out-of-equilibrium. If the buyer makes the jth offer, then the conclusion of the lemma follows trivially since $F_{j-1} = F_j$. Similarly, the conclusion of the lemma follows if the set k_j is empty. Suppose the seller makes the jth offer and k_j is nonempty. By Lemma 1, if $c' \in k_j$, then for all $c'' > c'$, $c'' \in k_j$. Therefore, if there exists a $c' \in k_j$ such that $c' < \inf\{\operatorname{supp} F_{j-1}\}$, then part (a) of (NS) applies and $\inf\{\operatorname{supp} F_{j-1}\} \leq \inf\{\operatorname{supp} F_j\}$. Otherwise, there is no $c' \in k_j$, such that $c' < \inf\{\operatorname{supp} F_{j-1}\}$ and the conclusion of the lemma must hold. Q.E.D.

The outcome of the bargaining process depends on the cost c of the seller and the buyer's initial beliefs F_0 regarding the seller's type. For arbitrary c and F_0 there are likely to be many sequential equilibria that satisfy (NS). Let $u(F_0, c)$ denote the seller's payoff in an arbitrary equilibrium. I am interested in the case where the buyer's beliefs are degenerate on a particular cost value since these are the sequentially rational beliefs when the bidding strategy is separating. This case is important because I wish to determine the bargaining outcome if the buyer can infer the seller's private information from his bid. Thus, I want to determine the properties of $u(\Phi_{\hat{c}}, c)$. To simplify notation, let $u(\Phi_{\hat{c}}, c) = \tilde{u}(\hat{c}, c)$.

Rubinstein (1982) shows that under complete information there is a unique subgame perfect equilibrium to the sequential bargaining game. Suppose that the buyer's value is v and the seller's cost is known to be c. Then the equilibrium initial offer of the buyer is $\sigma(v,c) = (\delta v + c)/(1+\delta)$, where $\delta = e^{-rz}$. If the initial offer is rejected, then all subsequent offers are equal to $(z, \sigma(\iota, \kappa))$ where ι is the value/cost of player making the offer and κ is the value/cost of the other player. The buyer will only accept offers less than or equal to $\sigma(c,v)$, and the seller will only accept offers greater than or equal to $\sigma(v,c)$.

Proposition 1. *Suppose $\tilde{u}(\hat{c},c)$ is the seller's payoff in a sequential equilibrium satisfying* (*NS*) *when the buyer's beliefs are degenerate on $\hat{c}$* (i.e., *initial beliefs are $\Phi_{\hat{c}}$*) *and the seller's cost is c such that $c, \hat{c} \in [\underline{c}, v]$. Then*

(i) *$\tilde{u}(\hat{c},c) = \sigma(v,\hat{c}) - c$, for all $c \leq \hat{c}$; and $\tilde{u}(\hat{c},c) \geq [\sigma(\hat{c},v) - c]\delta$, for all $c > \hat{c}$,*

(ii) *$\tilde{u}(\hat{c},c)$ is continuous in $\hat{c}$ at $\hat{c} = c$,*

(iii) *$\tilde{u}_1^+(c,c) = 1/(1+\delta)$ and $\tilde{u}_1^-(c,c) < 1/(1+\delta)$, where $\tilde{u}_1^+$ and $\tilde{u}_1^-$ denote the right- and left-hand derivatives with respect to the first argument.*

Before the proof of Proposition 1 can be presented I need the following lemma from Grossman and Perry (Lemma 3.1, 1986).[2]

Lemma 3. *Suppose that at offer j the buyer has beliefs F_j with $c_l = \inf\{\operatorname{supp} F_j\}$ and that the updating of beliefs is consistent with* (*NS*). *Then the buyer will accept any offer less than or equal to $\sigma(c_l, v)$.*

Proof of Proposition 1. Part (i): Believing that the seller has cost $\hat{c}$ the buyer will initially offer $\sigma(v,\hat{c})$, the initial offer in the complete information bargaining model. When $c \leq \hat{c}$, to change the buyer's beliefs, the seller would have to reject the initial offer and make a counter offer that makes him worse off. Since such an offer cannot lead to a higher payoff for sellers with cost $c \leq \hat{c}$, $\tilde{u}(\hat{c},c) = \sigma(v,\hat{c}) - c$.

To prove $\tilde{u}(\hat{c},c) \geq [\sigma(\hat{c},v) - c]\delta$, for all $c > \hat{c}$, it is sufficient to show that there exists an offer (Δ_2, p_2) that is accepted by the buyer and provides a payoff that is at least as high as $[\sigma(\hat{c},v) - c]\delta$. Notice that the offer $(z, \sigma(\hat{c},v))$ is accepted by the buyer (by Lemma 3) and results in a payoff of $[\sigma(\hat{c},v) - c]\delta$ to the seller.

Part (ii): To prove part (ii), it is sufficient to show that $\lim_{\hat{c}\to c^-} \tilde{u}(\hat{c},c) = \sigma(v,c) - c$. Suppose that $\langle t,p \rangle$ is an equilibrium outcome of bargaining with initial beliefs $\Phi_{\hat{c}}$ and actual cost $c > \hat{c}$. Then by part (i), $\langle t,p \rangle$ must satisfy

$$\sigma(v,\hat{c}) - c < \tilde{u}(\hat{c},c) = (p-c)e^{-rt} \quad . \tag{1}$$

It follows from part (i) and the fact that a seller with cost $\hat{c}$ can pretend to have cost c that

[2] While Grossman and Perry (1986) make a more restrictive assumption on how beliefs are updated, the conclusion of Lemma 2 is sufficient for this lemma to hold.

$$\sigma(v,\hat{c}) - \hat{c} \geq (p - \hat{c})e^{-rt} > (p - c)e^{-rt} \ . \qquad (2)$$

Inequalities (1) and (2) together imply that

$$\sigma(v,\hat{c}) - \hat{c} > (p - c)e^{-rt} > \sigma(v,\hat{c}) - c \ .$$

In the limit as $\hat{c}$ approaches c from below, $\sigma(v,c) - c = (p - c)e^{-rt}$.

Part (iii): The right-hand derivative is found by the differentiation of $\sigma(v,\hat{c}) - c$ with respect to $\hat{c}$. The proof of the result for the left-handed derivative is more complicated. Define the function μ as $\mu(\hat{c},c) = [\sigma(\hat{c},v) - c]\delta$ for $c \geq \hat{c}$. In the proof of part (i) I show that $\mu(\hat{c},c)$ is a lower limit on the seller's payoff when $c > \hat{c}$. Thus, $\mu(\hat{c},c) \leq \tilde{u}(\hat{c},c)$, for all $c,\hat{c} \in [\underline{c},v]$ and $c > \hat{c}$. Notice that $\mu(c,c) = \tilde{u}(c,c)$. Thus, for all $\epsilon > 0$ such that $c - \epsilon \geq \underline{c}$,

$$\frac{\mu(c-\epsilon,c) - \mu(c,c)}{-\epsilon} \geq \frac{\tilde{u}(c-\epsilon,c) - \tilde{u}(c,c)}{-\epsilon} \ ,$$

and hence, $\mu_1(c,c) \geq \tilde{u}_1^-(c,c)$. It can be shown that

$$\mu_1(c,c) = \frac{\delta^2}{1+\delta} < \frac{1}{1+\delta} \ .$$

Q.E.D.

4 Separating bidding strategies

I use Part (iii) of Proposition 1 to show the main result of this paper. This result shows that under (NS), the payoff function of the outcome from sequential bargaining is continuous and kinked at the point $\hat{c} = c$. The non-differentiability that occurs when bidding strategies are decreasing in the bidder's private information makes the incentive compatibility constraint impossible to satisfy. This is made precise by the following proposition.

Proposition 2. *Consider a strategy where the bidding strategies of the players are symmetric almost everywhere and decreasing functions of each bidder's private information. Then this strategy cannot be a sequential equilibrium strategy satisfying* (*NS*).

Proof. The proposition is shown by demonstrating that a decreasing bidding strategy cannot satisfy a necessary condition for an equilibrium.

Define π as

$$\pi(z,x_i) = [\beta_i(z) - c_P(x_i) + \lambda\tilde{u}(c_S(z),c_S(x_i))]G(z) \ .$$

The term within the brackets of the above expression is the payoff to bidder i given he wins the bidding with the bid $\beta_i(z)$. $G(z)$ is the probability that none of the bidders in $N\backslash\{i\}$ has a private information value greater than z. When all of the bidders follow bidding strategies that are symmetric almost everywhere and decreasing, probability $G(z)$ is the probability that bidder i wins the bidding with the bid $\beta_i(z)$. Therefore, $\pi(z,x_i)$ corresponds to the expected payoff of bidder i who bids $\beta_i(z)$.

If β_i is an equilibrium bidding strategy, the incentive compatibility constraint must hold. That is,

$$\pi(x_i, x_i) \geq \pi(z, x_i), \quad \text{for every } x_i, z \in \mathscr{X} \ . \tag{3}$$

Since β_i is a decreasing function, it is differentiable almost everywhere. Similarly, since Ψ and, thus, G are increasing functions, they are differentiable a.e. Hence for almost every x_i both G and β_i are differentiable. Thus, the incentive compatibility constraint implies $\pi_1^-(x_i, x_i) \geq \pi_1^+(x_i, x_i)$, must hold for almost all x_i. By part (iii) of Proposition 1 the left- and right-hand derivatives of $\tilde{u}$ exist, and hence, the left- and right-hand derivatives of π exist.

$$\begin{aligned}
\pi_1^-(x_i, x_i) =& [\beta(x_i) - c_P(x_i) + \lambda\tilde{u}(c_S(x_i), c_S(x_i))]g(x_i) \\
&+ [\beta'(x_i) + \lambda\tilde{u}_1^+(c_S(x_i), c_S(x_i))c_S'(x_i)]G(x_i) \\
\pi_1^+(x_i, x_i) =& [\beta(x_i) - c_P(x_i) + \lambda\tilde{u}(c_S(x_i), c_S(x_i))]g(x_i) \\
&+ [\beta'(x_i) + \lambda\tilde{u}_1^-(c_S(x_i), c_S(x_i))c_S'(x_i)]G(x_i),
\end{aligned}$$

where $g(x_i) = G'(x_i)$. Thus, condition (3) implies the necessary condition,

$$\tilde{u}_1^-(c_S(x_i), c_S(x_i)) \geq \tilde{u}_1^+(c_S(x_i), c_S(x_i)).$$

However, this violates part (iii) of Proposition 1. Q.E.D.

5 Conclusion

The result of Proposition 2 depends on the winning bid being observed. If the winning bid is observed, then when the bidding strategy is separating, the buyer can perfectly infer the seller's cost of the supplementary good. However, the outcome of sequential bargaining that follows is inconsistent with the existence of a separating bidding strategy in the first stage of the game, making separating equilibrium bidding strategies impossible. One possible solution to this problem is to implement bidding schemes where the winning bid is not observed by the buyer. Such schemes include open or oral auctions and sealed-bid second-price auctions where an independent mediator is employed to handle the bids and announce the outcome. Thus, if the auction designer finds separating outcomes advantageous, then we would expect to see such non-revealing auctions to be used in situations where second-stage bargaining could be affected by private information revelation in the first-stage.

Laffont and Tirole (1988) have shown that in repeated incentive contracts when the seller can make take-it-or-leave-it offers, perfect information revelation in the first stage of the relationship is impossible. In this paper I have shown that a similar result holds even when bargaining power is more equally distributed across the players. My proof relies on a number of restrictive assumptions. I consider a particular bargaining game and equilibrium refinement in the second stage, and I assume that there is no private information that is relevant to the second-stage bargaining that is not relevant to the first-stage auction (i.e., the buyer can infer the seller's expected cost of supplying the supplementary good after observing the winning bid). Similar assumptions can be found in Laffont and Tirole (1988). It may be the case that a weakening of one or more of these assumptions would cause the ratchet effect to vanish. Additional work is required to determine whether the ratchet effect is purely an artifact of these assumptions or if these results are examples of a more general phenomenon.

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