EFFICIENCY IN AUCTIONS WHEN BIDDERS HAVE PRIVATE INFORMATION ABOUT COMPETITORS

Ronald M. Harstad, Michael H. Rothkopf, and Keith Waehrer

ABSTRACT

The mainstream auction literature has emphasized the allocative efficiency of symmetric equilibria, in that the bidder with the highest willingness-to-pay wins for certain. This characterization is not robust to symmetric private information about the private information of rival bidders, as when a bidder in a timber auction may have imperfect private information about a rival bidder's inventory levels. We demonstrate the general difficulty of obtaining efficient outcomes in symmetric equilibria, under otherwise weak, mainstream assumptions. Furthermore, we find a restriction on the class of environments, a reasonable stylization of some auction markets, which suffices to yield efficiency in the Milgrom-Weber model of English auctions, but not in auctions with less complete information release.

Advances in Applied Microeconomics, Volume 6, pages 1–13.

ISBN: 1-55938-208-2

We define a more restrictive class of environments in which second-price but not first-price auctions attain efficiency.

I. INTRODUCTION

A priori asymmetric asset valuations are known to create the possibility of allocatively inefficient outcomes of auction mechanisms.[1] However, models of the isolated auction of a single, indivisible asset in an a priori symmetric environment have consistently exhibited allocative efficiency.[2] In all of these models, it is assumed that a bidder's private information can be summarized by a single scalar variable that is monotonically related to the bidder's asset value.[3] We extend these models by considering an environment where a bidder's private information is a vector. We show that efficiency in auctions is much less robust when private information is vector-valued. We also find sufficient conditions for the efficiency of English, second-price, and first-price auctions in such an environment.[4] These conditions indicate that because of its informational properties, it is easier to achieve efficiency when using an English auction than it is when first-price or sealed second-price auctions are used.

One consequence of the assumption of scalar-valued information that is monotonically related to asset value is that a bidder's equilibrium bid in a standard sealed bid auction fully reveals his private information. If the bidder with the highest private information value is always the bidder with the highest asset value, then standard sealed bid auctions are efficient in this type of environment. In many situations, however, it seems untenable to apply a model that implies that all of a bidder's private information is completely revealed by his bid. In this paper, we explore the efficiency of auctions in environments where bids cannot fully reveal a bidder's private information since the information is not summarizable by a scalar value.

In actual auction markets, a bidder's information that is relevant to the auction may not be summarizable by a single real-valued scalar. A bidder often knows (or at least believes he knows) additional information which is at least statistically informative about rival bidders' asset valuations or private information, and this knowledge is additional private information. A simple example of this can be specified by assuming that it is common knowledge that bidders 1 and 2 each privately observe a signal (X_1 and X_2) pertaining to their own asset valuation, but each may also privately observe a signal (S_{12} and S_{21}) pertaining to their rival's asset valuation. Hence, in this model each bidder's private information is vector-valued rather than a scalar.

Such a model might be appropriate when bidder 1 in a public-sector timber auction may have lost a privately negotiated placement of timber to bidder 2 recently, and bidder 2 has some other imperfect information correlated with the level of bidder 1's claimed but uncut timber inventory. Similarly, in an auto auction, used-car dealer 1 may drive past dealer 2's lot en route to the auction, and get a feel for whether dealer 2's lot is full of cars, or sparse; but he may not know which cars have been sold but not picked up; on the other hand, dealer 2 may drive by dealer 1's lot on the way to work in the morning, getting similar useful but incomplete information.

The relationship between these additional information variables and a bidder's asset valuation can take a number of forms. Bidder 1's private information about bidder 2 may be a noisy signal of X_2. In this case, learning what bidder 1 knows about X_2 would provide bidder 2 with no additional information about his own asset valuation.[5] Therefore, bidder 2 knows that bidder 1 knows something about what he already knows; bidder 2 just does not know what that something is. Another form this information can take is when bidder 1's private information pertaining to bidder 2's asset valuation is information that is unknown yet important to bidder 2. Hence, learning this information would cause bidder 2 to update his estimate of his own asset valuation. Therefore, bidder 1 knows something about bidder 2's asset valuation that bidder 2 does not know.

These forms of private information about rivals are introduced in the model below. When such private information is informative, symmetric equilibria of auctions may no longer be allocatively efficient even though a priori symmetry is maintained. The strength of assumptions needed to attain efficiency depends in some sense on the amount of information revealed during the auction and whether the price paid by the winning bidder depends on that information. Our results imply that the more information that is revealed during the auction, the easier it is to attain efficiency.

II. A GENERAL MODEL WITH INFORMATION ABOUT COMPETITORS

We define a set of environments, $\mathcal{E}$, that extends the set of affiliated-values environments[6] so as to allow for additional private information in the following way. Each bidder, besides receiving information "relevant" to his own value, observes information that "pertains to" the values of his rivals. Therefore, each bidder's private information is vector-valued rather than scalar-valued.[7]

Let the set of risk-neutral bidders be $\mathcal{N} = \{1, \ldots, N\}$. Assume each bidder i observes a private signal X_i of his own value for the asset. In addition, assume bidder i privately observes a vector $S_i = (S_{i1}, \ldots, S_{i\,i-1}, S_{i\,i+1}, \ldots, S_{iN})$. We interpret S_{ij} as information observed by bidder i regarding bidder j's value for the asset. (Notice that S_{ii} is not in the vector S_i; thus, S_i has dimension $(n - 1)$.) Let $S_{(i)} = (S_{1i}, \ldots, S_{i-1i}, S_{i+1i}, \ldots, S_{Ni})$ denote the information vector pertaining to bidder i's valuation that is privately observed by his rivals. Thus, S_i refers to information observed by bidder i about other bidders' values, and $S_{(i)}$ refers to the vector of information observed by other bidders about bidder i's value. Let $Z_i = (X_i, S_{(i)})$. Hence, Z_i is the vector of information variables that specifically pertain to bidder i's valuation of the asset.

The value of the asset to bidder i, V_i is determined by a continuous function v such that $V_i = v(Z_i, \{Z_j\}_{j \neq i})$. We assume that v is increasing in bidder i's information, X_i, and nondecreasing in its other variables. The random variables of the model, $(Z_1, \ldots, Z_N)$, are distributed according to a joint probability measure $\mathcal{P}$ on $\mathbf{R}^{N^2}$. Throughout, we assume $\mathcal{P}$ is atomless. The variables $(Z_1, \ldots, Z_N)$ are assumed to be affiliated. Affiliation implies that higher values for some variables make higher realizations of any other variables more likely (cf. Milgrom and Weber, 1982, pp. 1099 ff.).

Definition. An environment belongs to the set $\mathcal{E}$ if and only if the probability measure $\mathcal{P}$ is symmetric in $(Z_1, \ldots, Z_N)$ and symmetric in the elements of $S_{(k)}$ for each $k \in \mathcal{N}$ and the value function $v(Z_i, \{Z_j\}_{j\neq i})$ is symmetric in $\{Z_j\}_{j\neq i}$ and in the elements of $S_{(k)}$ for each $k \in \mathcal{N}$.

In an affiliated-values environment, bidder i only observes X_i, and his asset valuation is determined by a function that is increasing in X_i and nondecreasing and symmetric in the other bidders' private information, $(X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_N)$. Therefore, in an affiliated-values environment there are no additional private information values S_i.[8] Strictly speaking, the set $\mathcal{E}$ does not generalize the set of affiliated-valued environments because the private information in $\mathcal{E}$ environments are elements of $\mathbf{R}^N$ while the private information in affiliated-valued environments are elements of $\mathbf{R}$. However, there is a sense in which for any

affiliated-values environment it is possible to find an "equivalent" $\mathcal{E}$ environment. Such an $\mathcal{E}$ environment can be constructed from an affiliated-values environment by giving the bidders in the affiliated-values environment a vector of uninformative private information S_i. (If its distribution is degenerate, then S_i would be uninformative.) In such an $\mathcal{E}$ environment, the bidders' private information is vector-valued but the only relevant information is contained in X_i, and thus the presence of S_i makes no difference to a bidder's behavior. Therefore, first-price, second-price, and English auctions can be efficient for some environments in $\mathcal{E}$ since these auction forms are efficient in affiliated-values environments where the bidder with the highest X_i has the highest asset value.

We exploit the symmetry of the environments in $\mathcal{E}$ to focus on bidder 1. Let the supports of X_I and S_I be denoted $\tilde{X}$ and $\tilde{S}$ respectively. A *strategy* in a sealed-bid auction is a function $b : \tilde{X} \times \tilde{S} \to \mathbf{R}_+$, with $b(x,s)$ being the bid i submits when $X_i = x$ and $S_i = s$. An *equilibrium* is a strategy profile $(b_1, \ldots, b_N)$ such that b_i is a best response to the strategies $(b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_N)$ for all $i \in \mathcal{N}$. It is much more difficult to describe the strategy set in an English auction. The dynamic nature of an English auction makes it quite probable that a bidder's actions in the auction will vary by the information set that is reached in the course of the bidding.

III. EFFICIENCY IN AUCTIONS WHEN BIDDERS HAVE PRIVATE INFORMATION ABOUT COMPETITORS

A number of definitions of efficiency are possible in models of incomplete information such as auctions (see Forges, 1993; Holmstron and Myerson, 1983; Wilson, 1978). We opt for a *classical* or ex post definition of efficiency. An auction is said to be *allocatively efficient* (or simply efficient) if there exists an equilibrium such that a losing bidder's valuation $v(\cdot)$ never exceeds the winner's valuation. This definition of efficiency places a high burden on the information-summarizing properties of an auction. We will restrict attention to symmetric bidding strategies. Given the ex ante symmetry in the bidders, it would seem much more difficult (if not impossible) for an asymmetric equilibrium to achieve efficiency.

It is clear that first-price, second-price, and English auctions are efficient for some environments in $\mathcal{E}$ since, as described above, it is possible to find environments in $\mathcal{E}$ that are outcome-equivalent to affiliated-values environments. While it is possible to find environments in $\mathcal{E}$ that are efficient, it is also possible to find environments in $\mathcal{E}$ where no allocation mechanism or auction is efficient. The following remark formally demonstrates this point.

Remark 1. Consider an element of $\mathcal{E}$ such that $N = 2$, $v(Z_1, Z_2) = X_1 + S_{(1)}$, $\tilde{X} = \tilde{S} = [0,1]$, and $X_1, S_{(1)}, X_2, S_{(2)}$ are independently distributed. If there exists an allocation mechanism or auction that is efficient, then there must exist a *direct* mechanism with an efficient allocation as an equilibrium.[9] The following analysis demonstrates that no direct mechanism can be efficient for this environment and thus, no allocation mechanism or auction can be efficient. In this environment, a mechanism is efficient if $X_1 + S_{(1)} > (<) X_2 + S_{(2)}$ implies that bidder 1 is the winner (loser). Bidder 1's probability of winning in an efficient direct mechanism when he announces (x, s) is $\lambda(\alpha) = Pr\{X_2 - S_{(1)} < \alpha\}$, where $\alpha = x - s$. Bidder 1's expected value conditional on winning is $x + \delta(\alpha)$, where $\delta(\alpha) = E[S_{(1)} \mid X_2 - S_{(1)} < \alpha]$. Let $\pi(x, s)$ denote bidder 1's expected payment with announcement (x, s) in an efficient direct mechanism. Applying the incentive compatibility condition to an efficient mechanism, it is straightforward to show that if $x - s = \hat{x} - \hat{s} = \alpha$, then $\pi(x, s) = \pi(\hat{x}, \hat{s})$. Thus, define $\rho(x - s) = \pi(x, s)$. There exist $(x, s) \in (\tilde{X}, \tilde{S})$ and $(x', s') \in (\tilde{X}, \tilde{S})$ such that $x < x'$ and $\alpha = x - s > x' - s' = \alpha'$. Incentive compatibility implies,

$$[x + \delta(\alpha)]\lambda(\alpha) - \rho(\alpha) \geq [x + \delta(\alpha')]\lambda(\alpha') - \rho(\alpha')$$

$$[x' + \delta(\alpha')]\lambda(\alpha') - \rho(\alpha') \geq [x' + \delta(\alpha)]\lambda(\alpha) - \rho(\alpha).$$

The sum of the left-hand sides of the above expressions is not less than the sum of the right-hand sides. Therefore, $(x - x')[\lambda(\alpha) - \lambda(\alpha')] \geq 0$, which cannot hold since λ is an increasing function.

IV. A SUFFICIENT CONDITION FOR EFFICIENCY IN ENGLISH AUCTIONS

Suppose that observing $S_{(i)}$ is an imperfect substitute for observing X_i. Once X_i is observed, $S_{(i)}$ provides no additional information to bidder i about his own value for the object of the auction. Then no one who had access to the entire vector X would make any use of the vector S. This notion could be formalized in several ways, with differences that are inessential to our purposes. One could specify that any S_j is a garbling of X_{-j}, (cf. Milgrom and Weber, 1982, p. 1101) or that X_i is a sufficient statistic for (X_i, S_{jk}). We choose to use the following direct formalization:

Definition. An environment belongs to the set $\mathcal{E}_I$ if and only if it is in $\mathcal{E}$, the valuation function v is invariant with respect to $(S_1, \ldots, S_N)$, and $X_i > X_j$ implies $V_i \geq V_j$.

The assumption that $X_i > X_j$ implies $V_i \geq V_j$, which is included in the definition of $\mathcal{E}_I$, is a necessary and sufficient condition that first-price, second-price, and both forms of oral auctions being considered are efficient in affiliated-values environments. Notice that in an $\mathcal{E}_I$ environment the information contained in S_1 may still be of use to bidder 1 since at the start of the auction he does not know $X_2, \ldots, X_N$. Nonetheless, for a sufficiently restrictive and informative auction model, efficiency can be achieved for all $\mathcal{E}_I$ environments.

Theorem 1. An English auction is efficient for environments in $\mathcal{E}_I$.

Proof. Notice that the outcome is efficient if the Milgrom and Weber (1982) equilibrium strategies for an English auction are followed since in such a game the bidder with the highest X_i wins. Suppose bidders $2, \ldots, N$ adopt the Milgrom and Weber equilibrium strategy, denoted b^*.[10] Examine whether it is a best response for bidder 1 to adopt it as well. Bidder 1 cannot gain by deviating unless the deviation would change his behavior when he is one of the final two bidders. Suppose, without loss of generality, that only bidders 1 and 2 remain active in the auction. At this point in the auction, $X_3, \ldots, X_N$ can be identified by inverting strategies of bidders $3, \ldots, N$ since their exit prices are observed (recall that, by construction, b^* is degenerate in S_i). Let $x_3, \ldots, x_N$ be these inferred values.

Suppose that bidder 2's realization of X_2 is x_2. Since bidder 2 is assumed to follow b^*, he will quit at a price

$$p(x_2) = E[V_1 \mid X_1 = x_2, X_2 = x_2, X_3 = x_3, \ldots, X_N = x_N].$$

Suppose bidder 1's realization of X_1 is x_1 and S_1 is s_1. Then if he wins, his payoff is

$$E[V_1 \mid X_1 = x_1, S_1 = s_1, X_2 = x_2, \ldots, X_N = x_N]$$

$$- E[V_1 \mid X_1 = x_2, X_2 = x_2, X_3 = x_3, \ldots, X_N = x_N].$$

Notice that when b^* is followed, this payoff is positive if and only if bidder 1 wins the auction since for environments in $\mathcal{E}_1$

$$E[V_1 \mid X_1 = x_1, S_1 = s_1, X_2 = x_2, \ldots, X_N = x_N]$$
$$= E[V_1 \mid X_1 = x_1, X_2 = x_2, X_3 = x_3, \ldots, X_N = x_N]$$

for all s_I. Thus, b^* is a best response for bidder 1 since he can do no better than bidding so that he wins when this expected payoff is positive and loses when this expected payoff is negative. □

There is a clear sense in which the unusual extent of information revelation during the course of an English auction is essential to this efficiency result.[11] Even though English auctions and second-price auctions are similar in many ways (even outcome equivalent for $N = 2$), as is demonstrated by the following remark, there are environments in $\mathcal{E}_1$ in which second-price auctions are inefficient.

Remark 2. In an environment belonging to $\mathcal{E}_I$, bidder i would have the highest asset value if $X_i > X_j$ for all $j \in \mathcal{N} \setminus i$. Therefore, for a second-price auction to be efficient, bidder i 's equilibrium bidding strategy must depend only on the realization of X_i. Let $b^*(x)$ denote the equilibrium bidding strategy of a bidder with private signals (x, s). Let $\phi(x, y, s) = E[V_1 \mid X_1 = x, Y_1 = y, S_1 = s]$. The expected payoff of bidder 1 if he bids as if he had received signals (z, s) but actually received signals (x, s) is

$$\int_{\underline{x}}^{z} \{\phi(x, y, s) - b^*(y)\} f(y \mid s) dy,$$

where $Y_1 = \max\{X_2, \ldots, X_N\}$ and $f(\cdot \mid \cdot)$ is the density function of Y_I conditioned on S_1. The first-order condition for this objective function implies $\phi(x, x, s) = b^*(x)$ for all $x \in \tilde{X}$ and for all $s \in \tilde{S}$. Even though, for environments in $\mathcal{E}_1$, v does not depend on $S_1, \ldots, S_N$, affiliation implies that it is possible for $\phi(x, y, s)$ to be increasing in the vector s. In that case, the first-order condition for an efficient equilibrium to a second-price auction will not be satisfied. Unlike in an English auction, at the time the bid is submitted, a bidder's expected asset value conditional on winning, still depends on the vector s since the superior information contained in the vector $X_2, \ldots, X_N$ is unobserved and bidder 1's own value could depend on that information.

It is not just the static nature of second-price auctions that can prevent efficiency in this environment. Harstad and Rothkopf (1995) offer an "alternating recognition" model of English auctions in which, if two bidders are publicly competing, the price continues to rise, and other bidders are silent, not revealing whether they are still in competition. In that model, with high probability, the auction reaches prices at which only the highest two bidders are still competing (though they do not know this), and only a subset of rival bidders have revealed their information by public exits. Hence, the private information of some rivals is still unknown, and Remark 2 extends directly to alternating recognition auctions.

V. PRESUMPTIVE SUFFICIENT STATISTICS

Remark 2 shows the negative impact of private information about competitors. Nonetheless, the detachment of the price a bidder pays from his own bid, which is inherent in a second-price auction, is an efficiency-enhancing rule. This section demonstrates this claim by considering a much more restrictive set of environments. Let $W_1, \ldots, W_N$ denote the order statistics of $X_1, \ldots, X_N$ arrayed from highest, W_1, to lowest, W_N.

Definition. An environment is in $\mathcal{E}_2$ if and only if it is in $\mathcal{E}_1$ and the top two order statistics, (W_1, W_2), are sufficient statistics for the entire collection, $(W_1, \ldots, W_N)$.

Harstad and Levin (1985) found the closely related condition that W_1 is a sufficient statistic sufficient for dominance solvability of second-price auctions.

Theorem 2. A second-price auction is efficient for any environment in $\mathcal{E}_2$.

Proof. Let $\psi(x,y) = E[V_1 \mid X_1 = x, Y_1 = y]$, where Y_1 denotes the first-order statistic of $X_2, \ldots, X_N$. For affiliated-values environments Matthews (1977) and Levin and Harstad (1986) have shown that the unique symmetric equilibrium of the second-price auction is $b^*(x, s) = \psi(x, x)$. Since ψ is nondecreasing in both of its arguments, b^* results in an efficient outcome. We show that when b^* is followed by bidders $\mathcal{N} \setminus 1$, then b^* is a best response for bidder 1. For environments in $\mathcal{E}_2$, $\phi(x, y, s) = \psi(x, y)$, for $x \geq y$ and any s. If bidder 1 wins the auction, then his expected payoff is

$$\phi(X_1, Y_1, S_1) - \psi(Y_1, Y_1) = \psi(X_1, Y_1) - \psi(Y_1, Y_1)$$

which is positive if and only if $X_1 > Y_1$. Bidder 1 can do no better than win whenever $X_1 > Y_1$ which is the outcome when bidder 1 follows b^*. □

Corollary 1. An alternating recognition auction is efficient for any environment in $\mathcal{E}_2$.

Proof: The argument that a bidder should accept recognition when offered is unchanged from Theorem 1 in Harstad and Rothkopf (1995). The argument that it is an equilibrium for a recognized bidder to exit at price $\psi(x, x)$ follows exactly as in Theorem 2 above. □

In an alternating recognition auction, the final two bidders stochastically observe public exits of rival bidders, depending on whether the price rose above a rival bidder's willingness-to-pay before he was recognized. In environment $\mathcal{E}_2$, the information presumed in $\psi(x, x)$ is sufficient for a bidder's estimate of asset value, and he rationally ignores any private information regarding other bidders' values or any information inferred from public exits.

While second-price auctions are efficient for all $\mathcal{E}_2$ environments, the same is not true for first-price auctions. As is demonstrated by the following remark, there are $\mathcal{E}_2$ environments in which first-price auctions are inefficient.

Remark 3. An auction in an $\mathcal{E}_2$ environment is efficient only if bidder i wins whenever $X_i > X_j$ for all $j \in \mathcal{N} \setminus i$. Therefore, in an efficient first-price auction the common equilibrium bidding strategy b must be increasing in a bidder's signal about his own value and invariant with respect to his signals that refer to the other bidders' values. Thus, let $\beta(x) = b(x, s)$ on $\tilde{X} \times \tilde{S}$. A common equilibrium bidding strategy must also satisfy

$$\beta(x) = \arg\max_b \{E([\phi(x, y, s) - b] \cdot 1_{\{b > \beta(Y_1)\}} \mid X_1 = x, S_1 = s)\},$$

where 1_E takes the value 1 in event E and 0 otherwise. The first-order condition for $b(x, s)$ to be a best response requires that

$$[\phi(x, x, s) - \beta(x)] \frac{g(x \mid x, s)}{G(x \mid x, s)} - \beta'(x) = 0, \text{ for almost all } (s,x),$$

where g and G are the density and distribution functions of Y_1 conditioned on X_1 and S_1.[12] Strict affiliation implies that the ratio in the expression above is decreasing in s. For environments in $\mathcal{E}_2$, ϕ is constant with respect to s, yielding the contradiction.

VI. CONCLUSIONS

Assumptions about bidders' information play a critical role in auction models. In the past, the development of common-value and affiliated-values models radically changed the thinking that began with independent-private-values models (cf. Rothkopf, 1969; Capen, Clapp, and Campbell, 1971; Wilson, 1977, 1993; Milgrom and Weber, 1982; McAfee and McMillan, 1987; Rothkopf and Harstad, 1994). We have shown that a different aspect of the conventional assumptions about information, heretofore implicit, can be crucial. In particular, the assumption that bidders have no private information about rivals' private information is important. Indeed, it is essential to the allocative efficiency of commonly observed auction forms in a priori symmetric environments.

ACKNOWLEDGMENT

We gratefully acknowledge the financial support from National Science Foundation grants SES 91-08551 and SBR 93-09333. During the composition of this paper Keith Waehrer was a visiting assistant professor at Brigham Young University; he would like to thank them for their hospitality. Any opinions expressed in this paper are those of the authors and do not constitute policy of the U.S. Bureau of Labor Statistics.

NOTES

1. Maskin and Riley (1992) and Waehrer (1993) have analyzed first-price auctions when bidders are a priori asymmetric and find that the resulting allocations can be inefficient.

2. The efficiency characterization has a lengthy history, beginning with Vickrey (1961) for the first three auction mechanisms, assuming an independent-private-values environment, and risk-neutral bidders. Efficiency is trivially attained in a common-value environment with a reserve price and entry fee of zero. In the Milgrom and Weber (1982) affiliated-values environment, a sufficient condition for an efficient outcome is that the bidder with the highest private signal will have the highest value for the object of the auction ex post. With symmetrically risk-averse bidders, efficiency is implicit in Holt's (1980) analysis of an independent-private-values environment, and in Milgrom and Weber's model of an affiliated-values environment. For concreteness, we refer throughout to auctioning an asset for sale. The entire analysis has a complete and natural counterpart in an auction for procurement, in which allocative efficiency implies that the bidder with the lowest willingness to accept obtains the contract to supply.

3. All the models cited in the surveys by Wilson (1993) and McAfee and McMillan (1987) assume that private information is summarizable by such a scalar value.

4. Second-price sealed-bid auctions were first introduced to the literature by Vickrey (1961). The term "English auctions" is generally used to refer to ascending-value oral auctions. We consider two models of English auctions: Milgrom and Weber (1982) ("Japanese" auctions) and Harstad and Rothkopf (1995) ("alternating recognition" auctions). We use the term "English" auctions to refer to Milgrom and Weber's model of ascending-value oral auctions and use the term "alternating recognition" auctions to refer to the model of Harstad and Rothkopf.

5. However, a bidder could well find the information valuable in a strategic sense. Learning this information may help bidder 2 to outbid bidder 1.

6. See Milgrom and Weber (1982) for the first appearance of affiliated-values environments in the literature on auctions.

7. Since there is an isomorphic relationship between $\mathbf{R}^n$ and $\mathbf{R}$, the real difference between affiliated-valued environments and the set of environments we define also depends on how the information is related to a bidder's asset valuation.

8. In Milgrom and Weber's (1982) description of the affiliated-values model, there are information variables labeled S that can affect a bidder's asset value. However, those variables play a different role in their model than the S_i vectors in the present model. In Milgrom and Weber's affiliated-values environment, S represents information relevant to the bidders' asset valuations that is unknown to any of the bidders, for example, nonparticipant appraisals. When bidders are risk-neutral or in the absence of play following the bidding, this additional uncertainty plays no real purpose in the model. Thus, it is not included in the model presented here.

9. In a direct mechanism the allocation and payments are computed using the announcements of all of the bidders. Announcements are elements of $\tilde{X} \times \tilde{S}$. That is, the buyers announce their private information to the seller. If the mechanism satisfies incentive compatibility, then there is an equilibrium in which buyers will make truthful announcements. In this context, we use the concept of a direct mechanism not because we believe that direct mechanisms are used to allocate goods, but rather because if there is an allocation mechanism (including auctions) that in equilibrium results in an efficient allocation, then there must exist a direct mechanism with an efficient equilibrium. This observation is commonly referred to as the revelation principle (see Fudenberg and Tirole, 1991).

10. The bid function is defined in their equations (5) and (6), p. 1104.

11. There is even a sense in which efficiency of an English auction depends on the special form of English auction modeled by Milgrom and Weber (1982). We will show below that other ways of modeling ascending-value oral auctions (specifically, an alternating recognition auction) do not imply efficiency.

12. This is a straightforward adaptation of Theorem 14 in Milgrom and Weber (1982).

REFERENCES

Capen, E. C., R. V. Clapp, and W. M. Campbell. 1971. "Competitive Bidding in High-Risk Situations." *Journal of Petroleum Technology* 23(June):641–653.

Forges, F. 1993. "Some Thoughts on Efficiency and Information." In *Frontiers of Game Theory*, edited by K. Binmore, A. Kirman, and P. Tani. Cambridge: MIT Press.

Fudenberg, D., and J. Tirole. 1991. *Game Theory*. Cambridge: MIT Press.

Harstad, R. M., and D. Levin. 1985. "A Class of Dominance Solvable Common-Value Auctions." *Review of Economic Studies* 52(June):525–528.

Harstad, R. M., and M. Rothkopf. 1995. "An 'Alternating Recognition' Model of English Auctions." Mimeograph, Rutgers University.

Holmstrom, B., and R. B. Myerson. 1983. "Efficient and Durable Decision Rules with Incomplete Information." *Econometrica* 51(November):1799–1819.

Holt, C. A. 1980. "Competitive Bidding for Contracts under Alternative Auction Procedures." *Journal of Political Economy* 88(June):433–445.

Levin, D., and R. M. Harstad. 1986. "Symmetric Bidding in Second-Price Common-Value Auctions." *Economic Letters* 20:315–319.

Maskin, E., and J. Riley, 1992. "Asymmetric Auctions." Mimeograph, Harvard University.

Matthews, S. 1977. "Information Acquisition in Competitive Bidding Processes." Working Paper, California Institute of Technology.

McAfee, R. P., and J. McMillan. 1987. "Auctions and Bidding." *Journal of Economic Literature* 25(June):699–738.

Milgrom, P., and R. J. Weber. 1982. "A Theory of Auctions and Competitive Bidding." *Econometrica* 50 (September):1089–1122.

Rothkopf, M. H. 1969. "A Model of Rational Competitive Bidding." *Management Science* 15(March):362–373.

Rothkopf, M. H., and R. M. Harstad. 1994. "Modeling Competitive Bidding: A Critical Essay." *Management Science* 40(March):364–384.

Vickrey, W. 1961. "Counterspeculation, Auctions, and Competitive Sealed Tenders." *Journal of Finance* 16(March):8–37.

Waehrer, K. 1993. "Asymmetric Auctions with Application to Joint Bidding and Mergers." Working Paper #93-08, Department of Economics, Rutgers University.

Wilson, R. 1977. "A Bidding Model of Perfect Competition." *Review of Economic Studies* 44(October):511–518.

Wilson, R. 1978. "Information, Efficiency and the Core of an Economy." *Econometrica* 46(July):807–816.

Wilson, R. 1993. "Strategic Analysis of Auctions." In *Handbook of Game Theory*, edited by R. Aumann and S. Hart. Amsterdam: North-Holland/Elsevier Science Publishers.